Action-Reaction: Matter-Geometry interaction in GR

Adán Sus
Department of Philosophy
Universidad Autónoma de Barcelona
Spain

adansus@gmail.com

Abstract: In general relativity (GR) the so called response equations $T^{\mu\nu}{}_{;\nu}=0$ are a direct consequence of Einstein's field equations. From them one can derive the general relativistic version of an energy-momentum conservation law and a geodesic principle to the effect that the world lines of test bodies are the geodesics of the spacetime metric. It is well known that this result can be seen also as a consequence of the covariance properties of the theory: applying Noether's second theorem to the coordinate independent gravitational Lagrangian and using satisfaction of the gravitational field equations one gets precisely the response equations. This result can be generalised for metric theories of gravity that are Lagrangian based; in a 1974 paper by Lee, Lightman and Ni (LLN), they prove that if all the fields appearing in the Lagrangian are dynamical (varied in the action), $T^{\mu\nu}{}_{;\nu}=0$ is a consequence of the gravitational field equations.

Furthermore, there are other spacetime theories for which energy-momentum conservation cannot be seen as a consequence of the field equations but rather as a requirement that imposes certain restrictions on the covariance of the theories. Examples of this are unimodular relativity or the various field equations that Einstein tried before arriving at the final ones; as is known, at some point Einstein used energy-momentum conservation as a physical condition that would restrict the covariance of his sough-after field equations.

In this paper I explore whether this apparent difference between spacetime theories can help us in the search for a substantive notion of general covariance or a criterion that allow to distinguish GR from previous spacetime theories. An initial idea is that Noether's second theorem, through the LLN result, could provide a criterion to distinguish between formal and substantive versions of general covariance in the following sense: take a coordinate independent lagrangian spacetime theory and see whether the response equations are a consequence of field equations including fields other than the matter fields: if they are not, this is an indication of the presence of a non-dynamical spacetime structure. It turns out that this criterion is either still merely formal – if one understands that any condition obtained variationally can be considered a field equation – or ambiguous; I argue this by discussing the use of lagrangian multipliers to implement coordinate restrictions. Nevertheless, the criterion can be modified by demanding that the field equations for which $T^{\mu\nu}$ act as a source alone be sufficient to derive $T^{\mu\nu}{}_{;\nu}=0$. I discuss whether such a modification is enough to provide a substantive notion of general covariance.

I connect the search for this criterion with three interrelated conceptual issues risen from the interpretation of GR. First, this criterion can be seen as the realisation of an old metaphor about what distinguishes GR from previous spacetime theories, that in GR matter tells spacetime how to curve and spacetime tells matter how to move. In a more general fashion it could be formulated as an action-reaction approach to provide a notion of background independence: geodesic motion (at least for test bodies) is determined only through the field equations for which matter acts as a source. I look into the prospects of this approach and contrast it with the absolute objects program of Anderson and Friedman. Second, this attempt at giving content to the uniqueness of GR amongst spacetime theories can be linked to one of the virtues that Einstein, at some points, attributed to GR: namely, that the theory, contrary to Newtonian physics and Special Relativity, provides a dynamical explanation of inertia. I discuss different senses in which one can say inertia to be explained dynamically, using the criterion presented above, and look at whether according to them this is a differential feature of GR. Third, I relate the discussion to the question about the status of the equivalence principle(s) in GR.