

What about General Relativity Requires Interpretation?

Erik Curiel

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N.B.: though this submission has sections, it is only an abstract, not a complete paper.

If we are to surround ourselves with a perceptual world at all, we must recognize as substance that which has some element of permanence. We may not be able to explain how the mind recognizes as substantial the world-tensor $[R_{ab} - \frac{1}{2}g_{ab}R]$, but we can see that it could not well recognize anything simpler. There are no doubt minds which have not this predisposition to regard as substantial the things which are permanent; but we shut them up in lunatic asylums.

Arthur Eddington

The Mathematical Theory of Relativity, pp. 120–1

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1 What Requires Interpretation, and What Sort is Required?

Several eminent philosophers, mathematicians and physicists (for example, C. Rovelli, A. Connes, L. Smolin, C. Isham, D. Malament, J. Norton, J. Earman and G. Belot) have in recent years concluded that general relativity requires an interpretation. They arrive at seemingly the same destination having set out from diverse and varied starting points, motivated by problems ranging from the attempt to find a quantum theory of gravitation to the attempt to come to grips with the existence of generic singular structure in solutions to the Einstein field-equation. Some eminent workers in

the field (for example R. Geroch and R. Wald) do not see the need for such a thing, at least not in any substantive, interesting sense. In this paper, I will sketch a defense for the latter position.

The defense will come in two parts. In the first, I will assume as given an otherwise unarticulated sense of ‘interpretation’ that includes at least the idea that such a thing provides an explication of the concepts of one theoretical structure in the terms of another. (I think this is a minimal and uncontroversial demand to make of any sense of “interpretation”.) In the light of this assumed sense of the term, I will briefly rehearse two of the issues and problems advocates of the need for an interpretation tend to raise in coming to their conclusions, the Hole Argument and the invariance of the theory under the group of spacetime diffeomorphisms. In each case, I will conclude that, whatever else one may think of the importance of such problems and issues in their own right, none of them forces on us the need for a new theoretical structure in the terms of which we must interpret general relativity if we are to comprehend it. This should not come as a surprise, I think, for none of these problems and issues requires concepts or terms not already available in general relativity itself for their formulation. Along the way, I will point out analogous “problems” in other, non-quantum theories that *prima facie* do not point to a gap in our comprehension of these theories that only an interpretation of some sort could bridge, suggesting that the drive to find an “interpretation” for general relativity based on analogous problems may have nothing to do with the character of general relativity as a physical theory, but perhaps more to do with unresolved, purely psychological dissonances its conceptual structures leave us with.

In the second part, I will compare the situation for general relativity with that for quantum mechanics, a theory almost everyone agrees requires interpretative elucidation of some sort or other. I will focus on the problem of measurement. In quantum mechanics, nothing in the framework itself holds out any promise of answering the question: what is a measurement? This problem has two independent components, the combination of which lends the problem its depth and urgency. First, the question points to a lack of understanding of the production of models of physical systems in the theory; the production of such models, however, is the sole way that theoretical structures can accrue to themselves empirical content, and our comprehension of the way that such models represent physical systems is the sole guarantee we can have of the propriety and the accuracy of those models. Second, the question by its very formulation seems to require the introduction of extra-theoretical conceptual resources in order to come to grips with it, which is to say, one can appreciate the full import of the question only by way of a notion, “measurement”, that one cannot express in strictly quantum mechanical terms, perhaps not even in strictly physical terms.¹ None of the problems of general relativity raised by proponents of the need for an interpretation share these features.

¹Bell (1987) makes this point most forcefully and eloquently.

2 The Hole Argument

Einstein originally formulated the Hole Argument to highlight what he considered to be problems with his inchoate theory regarding what we would now call determinism.² Einstein saw it, initially, as a flaw in a covariant field theory that a distribution of ponderable matter sources would not serve by itself to fix the physical state (the “gravitational field”) of a 4-space that itself contains no ponderable matter but is bounded by the fixed ponderable matter. Consensus today seems to have it that the proper way of understanding the Hole Argument places it rather at the heart of the question about the ontic status of spacetime points. Its lesson, so claimed, is that one cannot identifiably individuate spacetime points in the absence of metrical structure, that there is no “bare manifold of points” under the metrical field.³

I think these two views of the force of the argument rest on the same implicit misapprehension of the role of ponderable matter in the initial-value formulation of the Einstein field-equation. Einstein’s initial understanding of the force of the argument is obviated by the observation that, if one takes a proper spacelike 3-slice in a spacetime and imposes appropriate initial data on it, then the metrical structure everywhere in that slice’s domain of dependence is determined, including in any “otherwise empty” hole regions. It is no more a problem that the distribution of ponderable matter by itself does not fix the state of spacetime than it is that the distribution of charges in Maxwell theory does not fix a unique solution of the Maxwell equations. The Maxwell field has a subsistence all its own, which informs and is in turn informed by the distribution of charges, but neither “causes” or “gives rise to” the other in any straightforward sense relevant to the physical comprehension of Maxwell theory.

Likewise, the current view of the force of the Hole Argument contains the implicit demand that the distribution of ponderable matter on its own suffice for the determination of all relevant physical structure in the spacetime manifold, including the structure of the manifold as a point-set. On this view, the metrical structure, and all other structure, must be derived from the distribution of ponderable matter. As in the Maxwell case, I think it makes more sense to say that the metrical structure has a subsistence, whatever exactly that may come to, on its own that informs and is informed by the distribution of ponderable matter. In some regions of spacetime, those we think of as occupied by ponderable matter, the curvature has non-trivial contributions from terms involving the Ricci tensor; in others it does not. This fact by itself does not allow one to privilege the metrical structure of one of those kinds of regions over that of the other when addressing questions of the existence of spacetime structure.

I believe that the misapprehension about the initial-value formulation has its roots in a particular way of understanding the semantics of the Einstein field-equation. Crudely put, I think the two views assume that the equation expresses a *causal* statement of the form, “ponderable matter gives rise to, determines, spacetime metrical structure.” I think a more helpful view of the equation can be had by looking at the analogous question of the relation between force and acceleration as expressed in Newton’s Second Law. Applied force, in Newtonian mechanics, does not “cause” or “give rise

²See Einstein (1914) and Einstein and Grossmann (1914) for two versions of the original argument, and Norton (1989, 1993) for historical and critical discussion.

³See, *e.g.*, Belot (1996) and Rovelli (1999).

to” acceleration in any deep sense. One may as well regard acceleration as just the measure of the applied force as one may regard the applied force as just the measure of the acceleration, with the coefficient of proportionality given by, respectively, the quantity of inertial mass and the inverse of the inertial mass. As Maxwell (1873) and Stein (1999) clearly explain, the attempt to view force as “giving rise” to acceleration in any causal sense profoundly misconstrues the relation between them. In the same way, metrical structure in general relativity constrains the distribution of ponderable stress-energy, as the distribution of ponderable stress-energy constrains the metrical structure. The theory by itself does not allow one to endow one of them with logical or metaphysical priority over the other.

I want to emphasize that my argument has no bearing at all on the question itself of whether or not spacetime points exist in some lofty or mundane sense. I claim only that, properly construed, the Hole Argument does not bear on this question. Indeed, I am far from convinced that the question of the existence of spacetime points has ever been well posed. What possible difference could an answer to it, one way or another, in the terms of any known or proposed theory of physics, make to the proper comprehension of the performance of an experiment or the proper construction of a model of a physical system?

3 Diffeomorphic Freedom

The group of diffeomorphisms of the spacetime manifold, in some sense or other, constitutes a group of symmetries of the physical phenomena treated by general relativity. Invariance of the Einstein field-equation under the action of diffeomorphisms, however, is not a true symmetry in the sense captured by Neother’s theorem, nor in the sense of gauge theory as employed in, *e.g.*, Yang-Mills theory, nor in any other well understood sense pertaining to physical theories. There has thus arisen the issue of what meaning this diffeomorphic freedom has.

A comparison is edifying. Diffeomorphic freedom underscores an irremediable mathematical ambiguity in the apparatus provided by general relativity for the modeling of experiments: the choice of the presentation of the spacetime manifold and metric one uses to model an experiment is fixed only up to diffeomorphism. Classical mechanics, as embodied in Lagrangian and Hamiltonian mechanics, shares similar ambiguities, slightly different in each formulation of the theory: the choice in Lagrangian mechanics of the Lagrangian, which determines the dynamical affine connection on the tangent bundle of configuration space; and in Hamiltonian mechanics the choice of symplectomorphism between the space of states and the cotangent bundle of configuration space, *i.e.*, the choice of symplectomorphic presentation of phase space. One feels no need to question the significance of the Lagrangian merely because one is free to choose its form more or less freely, as one is not driven to investigate the ontic status of points in phase space merely because one is free to choose whatever symplectomorphism one likes in its presentation. Manifestly, the choice of Lagrangian or the choice of symplectomorphism rests on nothing more than considerations of the type adumbrated by Carnap (1950) in his discussion of the choice of a linguistic framework for the investigation of philosophical and physical problems,⁴ considerations determined by what Geroch calls, somewhat

⁴This is not to say that I consider the choice of gauge in Maxwell theory to be a choice of Carnapian linguistic

archly and entirely aptly, “psychology”. It is clear in these cases that the existence of inevitable, more or less arbitrary, non-physical elements in the presentation of the models of a theory by itself does not require of one the provision of an interpretation of either Lagrangian or Hamiltonian mechanics. More to the point, it is clear in these cases that the physical significance of the theory’s models is not masked or polluted by the unavoidable arbitrariness in the details of their presentations. In the same way, the diffeomorphic freedom in the presentation of relativistic spacetimes does not *ipso facto* demand an interpretation, in so far as it in no way prevents us from focusing on and investigating what is of true physical relevance in systems that general relativity models. Perhaps not remarkably, quantum mechanics shares this sort of inevitable ambiguity—the Hilbert space of a quantum system does not determine its underlying “configuration space” any more than does the cotangent bundle in Hamiltonian mechanics.

Clarifying the sense in which the group of diffeomorphisms leaves structure invariant in general relativity is a delicate matter, but it does not follow from this delicacy that general relativity stands in need of an ‘interpretation’ in any interesting sense of the word.

4 Comparison to Quantum Mechanics

The theory that philosophers and physicists have before now felt most required an interpretation is quantum mechanics. With regard to the impetus for demanding an interpretation of a theory, the difference between general relativity on the one hand and quantum mechanics on the other could not be more complete: it is the difference between ambiguity in the machinery of the former theory that models experiments, and ambiguity as to what counts, according to the latter theory, *as* an experiment. Quantum mechanics requires at the end of the day a *deus ex machina* to arrive at a mathematical model of an experiment that offers definite predictions of the readings of measurement devices—in modeling a physical system, one must decide at what point in a system’s physical evolution the wave-function “collapses” in order to extract a definite prediction for the reading of a measuring apparatus, which is to say in more classical terms, one must decide what moment in the system’s interaction with its environment constitutes an “experiment”.⁵ General relativity and classical analytical mechanics, on the other hand, while offering up many superficially distinct ways of modeling any given experiment, do not require one to classify different moments in a system’s evolution as constituting or not an “experiment” in order to arrive at definite predictions for the readings of experimental apparatus—all moments are treated on an equal footing by the theories.

framework, only that the sorts of considerations that go into each choice are similar.

⁵Thermodynamics, in particular the Second Law, has a sort of ambiguity about it similar to that of quantum mechanics. Locally, it is not clear that the Second Law is even valid unless one is extremely judicious in picking one’s system of study, which includes selecting the interval of that system’s physical evolution, and *a fortiori* selecting the types of interactions the system will have with its environment, as the proper one(s) for the making of one’s thermodynamical measurements—which is to say that, so far as the Second Law is concerned, it is not clear what constitutes an “experiment” and what does not. Globally, it is not even clear that one can formulate a proper analogue to the Second Law. If we live in a universe, *e.g.*, that has a non-compact Cauchy surface or one that has none at all, then there is no guarantee that one will be able to formulate a cogent representation of “the total entropy of the universe at a given moment” to use in attempting to determine whether the total entropy of the universe is in some sense always non-decreasing.

Quantum mechanics demands an interpretation because it is not clear how to model physical phenomena, how to model the outcomes of experiments *simpliciter*: the predictions of standard quantum theory are in some sense in contradiction with the outcomes of experiments, but not in such a way as to invalidate the theory—an extraordinary state of affairs. There is no analogous problem in general relativity. We know how to model in the terms of the theory experiments that manifest and probe every phenomena suggested or predicted by the theory, with no inconsistency of any kind, for we know with no ambiguity what are the fundamental, physical terms and principles of the theory in which one articulates these models and draws conclusions on their basis. In a similar vein, the comprehension of special relativity’s dismissal of the idea of absolute simultaneity did not require an interpretation of the theory, in any sense of the term; it required only that investigators come to terms with the fact that the fundamental terms of the theory does not allow for the rigorous, physically relevant articulation of the fundamental terms of Newtonian physics. In quantum mechanics, we do not even know what the fundamental terms and principles—‘measurement’? ‘observable’?—ought to be.

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