<u>**Whitehead's Interpretation of Einstein's General Theory of Relativity**</u> **R. Desmet**

§1 Introduction

Alfred North Whitehead (1861-1947), contrary to Helmholtz, and in agreement with Poincaré, held that the metric of physical space cannot be determined by physics. Consequently, even though he admired "the magnificent stroke of genius by which Einstein and Minkowski assimilated time and space" (**R** 88) in the Special Theory of Relativity (STR), he did not agree with Einstein's Helmholzian interpretation of the metric of space-time in terms of rigid rods, periodic clocks, and the constant transmission of light. According to Whitehead, rigidity, periodicity, and constancy, already presuppose the natural metric rooted in our recognition of congruence. Hence, in **PNK** and **CN**, Whitehead – without rejecting the geometry of STR, let alone the physics – set out to show that Minkowski's space-time can be reinterpreted by abstracting it from our spatio-temporal sense-perceptions, and by showing that the metric of STR is in fact the natural one.

However, our point of departure is not STR, but Einstein's 1916 General Theory of Relativity (GTR), epitomized by the following well-known equations:

(1) $$R_{ik} - \frac{1}{2} g_{ik} R = -\kappa T_{ik} \text{ with } \kappa = \frac{8\pi\gamma}{c^2}$$

(2) $$ds^2 = g_{ik} dx^i dx^k$$

(3) $$\delta \int ds = 0 \text{ and } ds^2 = 0$$

The point of Einstein's theory is to calculate by means of his law of gravitation (1) the fundamental tensor g_{ik} from the mass-energy distribution tensor T_{ik}. The fundamental tensor represents both the space-time metric and the gravitational field. The metric is determined by (2). The path of a mass-particle is calculated by the first, and the path of a light-ray by the second of the laws of motion in (3).

Whitehead was seriously challenged by GTR, especially after the 1919 confirmation of its prediction concerning the path of star-light passing the Sun. In GTR, the space-time manifold can no longer be considered as uniform and natural. It is variably curved by the distribution of mass and energy, and – even though GTR passed "the narrow gauge" of scientific experiment (**R** 4) – it split off from the general character of our sense-experience. Consequently, GTR implies "confusion." (**R** 83) E.g., how can the mass-energy distribution across space be given as input for solving (1), if space is only known *after* solving (1)? It seems to presuppose the metric it is supposed to define, and the pre-relativistic Poincaré critique of Helmholtz resurfaces again. Also, the bifurcation of space-time into 1°) the *really* heterogeneous space-time of the GTR, and 2°) the *apparently* uniform space-time of our common sense-experience, implies a confusing bifurcation between: 1°) our physical *theory* (e.g., calculating the local consequences of varying space-time curvature, such as the changing impact of star-light on photographic plates caused by varying earth-moon-sun-star-configurations) and our common measurement *practice* (e.g., of picturing the global configuration in our visual space, including the deviation of the path of star-light from the straight path, thereby violating the theory that the path of light-rays is not really bent in our uniform visual space, but that space itself is "bent").

In **R**, Whitehead rejected the heterogeneous space-time manifold of GTR, arising from the confusing identification of the physics of gravitation with the geometry of space-time, but he did not reject the physics of GTR as such, for he had "no doubt as to its general correctness." (**R** 59) Hence, Whitehead's "alternative rendering of the theory of relativity [maintains] the old division between physics and geometry. Physics is the science of the contingent relations of nature and geometry expresses its uniform relatedness." (**R** v-vi) More specifically, Whitehead's alternative theory of gravitation presupposes the Minkowskian space-time manifold of STR because of "its consonance with the general character of our direct experience." (**R** 4) Furthermore, "it also presupposes the general method of seeking tensor or invariant [...] laws of the physical field, a method due to Einstein" (**R** 88). And, of course, its aim is to reformulate Einstein's tensor law of the contingent phenomena of gravitation against the uniform background of the presupposed metric.

The result of Whitehead's attempts to reach that aim, his 1922 alternative theory of gravitation, is epitomized by the following equations:

(4) $$dG^2 = c^2dt^2 - dx^2 - dy^2 - dz^2$$

(5) $$dJ^2 = dG_m^2 - \frac{2}{c^2}\sum_M \psi_M dG_M^2 \text{ with } \psi_M = \frac{\gamma M}{\left(1-\frac{v_M^2}{c^2}\right)^{\frac{-1}{2}}\left(r - \frac{\vec{v}_M \cdot (\vec{x}_m - \vec{x}_M)}{c}\right)_{ret}}$$

(6) $$\delta\int m\sqrt{dJ^2} = 0 \text{ and } dJ^2 = 0$$

The point of Whitehead's theory is that geometry and physics are not identified in one fundamental tensor g_{ik}, but are differentiated by two tensors: the Galilean tensor G_{ik} and the potential impetus tensor J_{ik}. The former tensor determines the Minkowskian metric (4), known from Einstein's STR. The latter tensor is calculated by means of Whitehead's law of gravitation (5) in terms of (4) and in terms of the *retarded* potentials ψ_M associated with the masses M which are causally correlated with a mass m by the relation $c(t_m - t_M) = r$ [which, by the way, explains the *ret*-label in (5)]. The path of m and the path of a light-ray are calculated by the two laws of motion in (6).

Whitehead's formulae (4) and (6), representing the metric and Whitehead's laws of motion, are comprehensible by comparison with the metric of STR and with Einstein's laws of motion. Formulae (5), representing Whithead's law of gravitation, however, is the one that needs explanation and comparison with Einstein's law of gravitation. Unfortunately, all explanations of Whitehead's law of gravitation deal with his law in isolation from the context of British physics in the decade prior to the 1922 publication of **R**. Breaking with this habit, I will explain it against the background of the relativity work of two physicists who crossed Whitehead's path: Ebenezer Cunningham (1881-1977) and Ludwik Silberstein (1872-1948). Contrary to Whitehead, both these men have received due attention from historians of physics. (See, e.g., with regard to Cunningham: Goldberg 1970:113-117 & Warwick 2003:399-442; with regard to both: Sanchez-Ron 1987:39-53 & 1992:59-72; and with regard to Silberstein: McCausland 1999:278-283 & Duerbeck & Flin, 2005:186-209 & 2006:1087-1094)

§2 Historical Interlude

When dealing with Whitehead as an applied mathematician, the first aspect to take into account is the influence of Maxwell and his British followers, especially J. H. Poynting, J. J. Thomson (not to be confused with William Thomson, the later Lord Kelvin), and Joseph Larmor. These three physicists were the principle Cambridge members of the second generation of Maxwellians [G. F. FitzGerald, Oliver Heaviside, and Oliver Lodge, constituting the first generation of British Maxwellians (cf. Hunt 1991:2)]. Poynting, Thomson, and Larmor, were typical Cambridge products. Poynting did his Cambridge Mathematical Tripos exam in 1876, and Thomson and Larmor in 1880; all three were coached by Edward Routh (who excelled during the Tripos examination of 1854, and beat Maxwell into second place); and all three attended the intercollegiate courses on Maxwell's 1873 *Treatise on Electricity and Magnetism*, given by Maxwell's friend W. D. Niven. (Cf. Warwick 2003:333-398)

Whitehead was a near contemporary of Poynting, Thomson, and Larmor, and he was a similar Cambridge product: he did his Cambridge Tripos exam in 1883; he was also coached by Routh; he also attended Niven's courses; and, being slightly younger, he attended Thomson's lectures on electromagnetism. (Cf. Lowe 1985:92-109) Moreover, Bertrand Russell also reminds us that "Clerk Maxwell's great book on electricity and magnetism [was] the subject of Whitehead's Fellowship dissertation," and that "on this ground, Whitehead was always regarded at Cambridge as an applied, rather than a pure, mathematician." (Russell 1959:33) Whitehead often refers in his work to Maxwell, but hardly to Poynting, Thomson, and Larmor (see however, e.g., **ESP** 334, **PNK** vi, 42, 159, **CN** 151, **R** 5), and yet, an attentive reader of Whitehead's work can see multiple traces of the influence exercised on Whitehead, e.g., by Poynting's theorem on the energy flows in the electromagnetic field, by Thomson's and Larmor's electronic theory of matter, and by Larmor's adherence to the principle of least action.

According to Whitehead's biographer (Lowe 1990:6-14), in June 1911 Karl Pearson vacated the Goldschmidt chair of Applied Mathematics and Mechanics at University College, London, and his assistant Ebenezer Cunningham was asked to continue his teachings prior to naming a final successor. In July 1911, however, Cunningham was already released to accept a lectureship at Cambridge, and Whitehead – who had moved from Cambridge to London in 1910, and was in search for a job – gladly accepted to replace Cunningham during the interregnum year 1911-1912. Whitehead hoped to be the final successor of Pearson, but mid March 1912, his hopes were destroyed when he learned of the appointment of another applied mathematician (L. N. G. Filon). Whitehead tried to turn the tide and wrote a letter to the Provost of University College on March 16, 1912, in which we can read the following revealing sentence: "During the last twenty-two years I have been engaged in a large scheme of work [which] had its origin in the study of the mathematical theory of Electromagnetism […]." Whitehead stayed at University College during the years 1912-1913 and 1913-1914, occupying a chair in pure mathematics, but then left it for the Imperial College of Science and Technology, where he secured a professorship in applied mathematics.

According to Silberstein's biographers (Flin & Duerbeck 2006:1087-1089), this physicist from Polish origin, German student of, e.g., Helmholtz and Planck, and Italian lecturer in mathematical physics, moved from Italy to London in 1912, where he obtained a lectureship at University College London.

These historical details are relevant because they show that Whitehead was not only a pure, but also an applied mathematician, and that the authors of the two first British books on STR crossed Whitehead's path.

Cunningham was twenty years younger than Whitehead, but a Cambridge product as well: he went to St. John's College (Whitehead to Trinity College), and was coached by R. R. Webb (Whitehead by Routh). To stress the importance of the impact of the Cambridge coaching on the work of Cunningham (and Whitehead) which I will discuss in §3 (and §4), I quote a detail from Andrew Warwick's *Masters of Theory:* "For example, […] coaches such as Routh and Webb spent a good deal of time drilling their pupils in the solution of Laplace's equation. The reason for this was that the concept of 'potential' was one that could be applied not just to gravitational theory and the various branches of mechanics but to thermodynamics, electrostatics, magnetostatics, and electromagnetism. Having learned a range of techniques for solving problems in, say, gravitational potentials, a coach could introduce a new topic such as electrostatics by developing the analogy between gravitational and electrostatic potential theory. Likewise Lagrangian dynamics and the principle of least action were powerful methods precisely because, although dynamical in origin, they could be applied to numerous physical phenomena […]." (Warwick 2003:278)

At St. John's, Cunningham was a student of Larmor. Alongside his 1903-1904 research in the theory of differential equations, he made a long and careful study of Larmor's *Aether and Matter,* and a few years later he came across Einstein's 1905 paper "On the Electrodynamics of Moving Bodies." Inspired by Larmor's Electronic Theory of Matter (ETM) and by Einstein's contribution to it – according to Cunningham, above all a very powerful mathematical technique – Cunningham began his own work on ETM and STR, and his involvement culminated in his 1914 monograph, *The Principle of Relativity.* (Cf. Warwick 2003:409-414)

Silberstein's first investigations on STR started in 1911, when he published a quaternion-formulation of STR. His "course of lectures" on STR, "delivered in University College, London, 1912-1913," developed into his 1914 monograph, *The Theory of Relativity.* (Silberstein 1924:v) Silberstein developed into a passionate, but also highly critical adherent of Einstein's theories of relativity. E.g. in 1918 he wrote an article on "General Relativity without the Equivalence Hypothesis," and in 1919 he pronounced his scepticism about Eddington's results and interpretations of the solar eclipse observations set-up to verify GTR.

The evidence that Whitehead knew Silberstein and his writings is quite strong. During the academic year 1912-1913, Whitehead and Silberstein were University College colleagues.

In 1915, Whitehead joined the Aristotelean Society, and frequented Percy Nunn. (Lowe 1990:90) Nunn was close with Silberstein. He read the proofs of Silberstein's 1914 book (Silberstein 1924:v), and he wrote to Lord Haldane that he "mixed a good deal with men like Silberstein, who are keen followers and even developers of the theory of relativity when it first came among us." (National Library of Scotland, Haldane Archive, MS 5915 / folio 192) On the other hand, Nunn and Whitehead became close friends. (Lowe 1990:161 & 240) Nunn exercised considerable influence on him (**PNK** viii & **PR** xii), and their relationship was one of the reasons why some philosophers associated Whitehead with the New Realism of which Nunn was a proponent (Lowe 1990:102 & 109). No wonder that when Nunn published his own relativity book – *Relativity and Gravitation* – both Silberstein and Whitehead are repeatedly mentioned. (E.g., Nunn 1923:6-7) In April 1919, Whitehead explicitly acknowledged that he "received suggestive stimulus from Dr. L. Silberstein's *Theory of Relativity*." (**PNK** vii) In November 1919, when Silberstein questioned Eddington (during the famous joint meeting of the Royal Society and the Royal Astronomical Society on the May 1919 solar eclipse observations), Whitehead was present. And finally, in 1923, G. Temple read his paper "A Generalisation of Professor Whitehead's Theory of Relativity" at the Physical Society of London: in the presence of Whitehead, Temple explicitly treated Silberstein's 1918 article as a precursor of Whitehead's theory.

The evidence that Whitehead knew Cunningham's work is not equally strong, but given the fact that the former succeeded the latter in 1911; given their common interest in ETM and STR; given their teaching curricula; and finally, given the status Cunningham's 1914 book acquired; it is almost excluded that Whitehead did not read Cunningham's *The Principle of Relativity*. I hold it as more than a sheer coincidence that Whitehead's 1922 book has the exact same title.

Anyway, it is a reasonable working hypothesis to presuppose that Whitehead knew not only Silberstein's pioneering 1914 monograph on STR, but also the one written by Cunningham, as well as Silberstein's 1918 article on "General Relativity without the Equivalence Hypothesis." Consequently, I will now list some topics in these writings that may have guided Whitehead when developing his own theory of gravitation.

§3 Silberstein and Cunningham

As said, Whitehead's aim was to separate the geometry of space-time, which he considered to be Minkowskian, from the physics of gravitation. However, he also wanted to retain Einstein's pioneering method of formulating laws as tensor invariant expressions, to have Newton's law of gravitation as an approximation, and to account for the gravitational phenomena associated with Einstein's success: the 1.7" deflexion of a ray of light going past the sun, the residual 43" per century rotation of the orbit of Mercury unexplained by Newtonian calculations, and the displacement of the spectral lines of light reaching us from the surface of large stars towards the red end of the spectrum. (Cf. Einstein 1916:160-164) Hence, Whitehead listed three requirements: "These requirements are, (i) to have no arbitrary reference to any one particular [reference-]system, and (ii) to give the Newtonian term of the inverse square law, and (iii) to yield the small corrections which explain various residual results which cannot be deduced as effects of the main Newtonian law." (**R** 85)

That Whitehead formulated a theory of gravitation fulfilling all these requirements, is less of a miracle than might appear at first sight. The study of Silberstein's 1918 article may have been an important source of inspiration. Its first sentence reads: "The generalized theory of relativity […] has one very strong point, the requirement of general covariance of all physical laws, and one weak point […] the so-called 'equivalence hypothesis' which places gravitation on an entirely exceptional and privileged footing, bringing it into intimate connection with the line-element of the world." (p.94) Silberstein noticed "the importance and utility of the requirement of general covariance" (p.96), and he added that "if a physical law is written entirely in tensors, it will retain its form in passing from one system of reference to any other." (p.98) But Silberstein's aim was *"to emancipate the fundamental tensor […] from the influence of gravitation."* (p.100) So, he proposed "to reject the gravitational 'equivalence hypothesis,' but to retain the postulate of general covariance of physical laws." (p.100)

Silberstein assumed "the fundamental tensor g_{ij} [to be] *always, under all circumstances […] essentially the same"* (p.100), and he claimed "for our world *a constant curvature* […] denoted by $k = 1/R^2$." (p.101) Silberstein's corresponding line-element or metric "can be written, as is well known, in polar coordinates [as] $ds^2 = c^2dt^2 - dr^2 - R^2 \sin^2 \frac{r}{R}\left(d\phi^2 + \sin^2\phi d\theta^2\right)$ " (p.102) Most important in his paper is Silberstein's example of a covariant law of motion of a particle, the description of which he started as follows:

"The equations of motion of a free particle are contained in $\delta\int ds = 0$, the variational equation of the world-geodesics. And the idea easily suggest itself to derive possible laws of motion of a non-free particle (or one 'acted on by external forces') from similar variational equations after an appropriate amplification of the integrand. The purpose of the present section is to give only a very simple *example* of a generally covariant law of motion obtainable by this method […].” (p124)

Silberstein replaced the integrand ds with $ds - 2\Phi ds$ where Φ is a scalar function, and stated that “the laws of motion [are] embodied in an equation of the form $\delta\int(1-2\Phi)ds = 0$ [where] Φ plays the part of a scalar *potential* of the ‘force’ soliciting the particle” (p.124-125), and where Φ – in the case of “a space of constant curvature” (p.126) – satisfies the differential equation

$$(7) \qquad \nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -4\pi\rho$$

Silberstein added: “The scalar ρ is to be considered as some given function of the four variables. The ‘potential’ Φ is thus propagated, in any natural system, with light velocity c […]. If $\rho \neq 0$ within a certain region, then apart from waves (satisfying the reduced equation) Φ can be represented as the retarded potential of that distribution, or it can be treated by the well-known four-dimensional method. […] As has already been said, the above is intended merely as an example of generally covariant laws of motion. Yet, after all [equation (7)] may turn out to be helpful in describing gravitation.” (p.127)

Equation (7) is a famous equation in physics, because if ρ is taken to be the charge density, it is d’Alembert’s equation determining the potential in electrodynamics, and if $\rho = 0$ then it is the wave equation. Turning from electrodynamics to electrostatics, yielding $\frac{\partial^2\Phi}{\partial t^2} = 0$, d’Alembert’s equation turns into Poisson’s equation and the wave equation turns into Laplace’s equation. So Silberstein suggested to replace Einstein’s approach with one in which gravitation is described by means of a scalar potential similar to the one used in electrodynamics, and the law of motion is expressed in terms of a constant metric and this gravitational potential.

This brings us smoothly to Cunningham’s 1914 book. Actually, Cunningham was an expert on the theory of differential equations. For his initial research on the topic in Cambridge, he was awarded a Smith’s prize in 1904. (Cf. Warwick 2003:410), and his interest in conformal transformation techniques to solve equations led, e.g., to his 1908 paper on the “Conformal Representation and the Transformation of Laplace’s equation.” (Cf. Warwick 2003:418-422) Cunningham had a lot to offer for anyone interested in equation (7), d’Alembert’s equation determining the electrodynamic potential. But, there’s more.

Cunningham's interest in transformation techniques was inspired by his reading of Larmor and Einstein, and conversely, because his main interest was the ETM and STR, he applied his mathematical skills, e.g., to the treatment of the electrodynamic potential of the electron in terms of Minkowski's four-dimensional calculus. The electrodynamic potential can be written as a four-dimensional vector $\mathbf{A} = (A_1, A_2, A_3, i\Phi)$ where $\vec{A}$ and Φ are the ordinary vector and scalar potentials in terms of which the electromagnetic field can be expressed, and where each of the four components of the four-dimensional vector satisfies equation (7). In the case of a single electron – a point-charge – the charge density $\rho = 0$, except in the point-source of the electromagnetic field itself. How to express the four-dimensional potential in that specific case? Let us turn to Cunningham's 1914 book.

In Chapter IX, §6, on "Retarded potentials," Cunningham stated that the "values of the ordinary vector and scalar potentials have been given by Liénard and Wiechert" in 1898 and 1900 respectively. (p.108) Cunningham next expressed these potentials as a four-dimensional vector. The four-dimensional potential **A** of the electron-charge *E* at point $\vec{x}_e$ and time t_e can be completely captured by means of **A** = $\Omega_E\left(\frac{v_{E,1}}{c}\Phi, \frac{v_{E,2}}{c}\Phi, \frac{v_{E,3}}{c}\Phi, i\Phi\right)$ with

$\Omega_E = \left(1 - \frac{v_E^2}{c^2}\right)^{\frac{-1}{2}}$ and $\Phi = \frac{E}{\Omega_E\left(r - \frac{\vec{v}_E \cdot (\vec{x}_e - \vec{x}_E)}{c}\right)_{ret}}$, and where the subscript *ret* stands for the condition $c(t_e - t_E) = r$.

Following this rewriting of the Liénard-Wiechert potential, Cunningham showed that his expression for **A** could be conceived in terms of two 4-vectors, **V** = $\Omega_E\left(\frac{v_{E,1}}{c}, \frac{v_{E,2}}{c}, \frac{v_{E,3}}{c}, i\right)$ and **R** = $(x_e - x_E, y_e - y_E, z_e - z_E, ic(t_e - t_E))$. Indeed, **A** can easily be written as the charge *E* multiplied with **V** and divided by the inner product of **V** and **R**. And Cunningham concludes: "The fact that **A** is a 4-vector, and hence the relativity of the potentials of Liénard and Wiechert is now clearly shown by the fact that the factor **V** in the nominator is a 4-vector while the factor **V.R** in the denominator is an invariant." (p.109)

His conclusion on the Lorentz invariance of the denominator, taken on itself, might not seem very relevant, but Cunningham linked his 4-vector expression in a footnote to his "work founded on that of Poincaré for modifying the law of gravitation to conform the Principle of Relativity" (p.109), work which he treated in Chapter XIII, §§10-16. Turning to that Chapter, and hence, from electromagnetism to gravitation, we learn that "in order to eliminate a discussion of the mode of transmission of gravitation we must *assume a law of propagation,"* which Cunningham took to be "the relation $r = c(t_1 - t_2)$ where r is the distance between the positions of the particles at times t_1, t_2 respectively; that is, we assume the influence of gravitation to be propagated with the velocity of light." (p.174) Cunningham proceeded to introduce a number of 4-vectors and products of 4-vectors, namely, the ones he judged to be "the simplest and the most fundamental." (p.175) Two of his preferred 4-vectors are **V** and **R**, and one of his preferred products is **V.R**, the product he used in Chapter IX as the denominator of his Liénard-Wiechert expression.

Most importantly, Cunningham proceeded with a proposal on how to modify the law of gravitation in the context of STR. First he wrote a quite general "invariant equation of gravitational motion" (p.176) in terms of all his preferred 4-vectors and products. But, after discussing the general equation, he concluded with the reduction of the general equation to two closely related particular equations [denoted (I) and (II)], both in terms of **V.R**, and both deriving from the 1911 work "done by Professor de Sitter," the famous Dutch astronomer. Cunningham remarks that one of the conclusions of de Sitter was that equation (II) – the second of the two particular equations – "would lead to a secular motion of the perihelia of the planets which in the case of Mercury amounts to about 7" per century," and Cunningham adds: "An effect of this kind has for some time been known by practical astronomers to exist, though the magnitude known is about 40" per century. Various hypothesis have been suggested to explain it. One of them proposed by Gerber in 1898 quite independently of the principle of relativity is the possibility that the Newtonian Law of Gravitation is only approximate, and that the more accurately gravitational influence is propagated with the velocity of light, and that a correction of nature very similar to that suggested by equation (II) must be applied to the usual expression for the force on the planet. He arrives at the conclusion that the known motion of the perihelia can be so explained." (p.180)

Cunningham's reference to Gerber, and Gerber's consistency with the observed secular motion of the perihelion of Mercury, are relevant. Poincaré's 1906 approach (to which not only Cunningham refers, but Silberstein as well 1914:86), de Sitter's 1911 approach (Cunningham 1914:178-180), and Silberstein's 1918 approach (cf. Temple 1923:177), were *all* inconsistent with the sought for 43" per century additional precession of Mercury's perihelion. Only Einstein's 1916 GTR seemed consistent with it. However, Gerber's independent 1898 approach showed that it was not impossible to arrive at the correct value with an alternative approach. Moreover: Gerber's method was also "based on the assumption of a retarded gravitational potential!" (McCausland 1999:283)

I will now give a heuristic derivation of Whitehead's law of gravitation, guided, *not only* by his philosophical intuitions and by Einstein's STR and GTR (let's not forget that his theory was *ad hoc*), *but also* by the information listed above. Whitehead, being a professor of applied mathematics, even taught courses on "the theory of the potential and of attraction" – e.g. during the academic year 1911-1912 he replaced Cunningham! (Lowe 1990:7) – as well as on relativity – e.g. during the academic year 1919-1920, when he inspired and encouraged his student Herbert Dingle to write *Relativity for All* (Lowe 1990:65 & Dingle 1922:vi). So he must have absorbed the information listed above from his study of applied mathematics, including his reading of Silberstein and Cunningham.

Given Whitehead's interest in separating the metric of GTR from the physics of GTR, in the unique status of the metric of STR, and in ETM, he must have concluded from his study of applied mathematics that the best way to proceed was:

1) to formulate a law of gravitation, not only in terms of the preferred Minkowskian metric, but also in terms of an invariant retarded potential of gravitation, associated with a mass density ρ and satisfying Silberstein's d'Alembertian equation (7), or better even, associated with a distribution of discrete point-masses *M* so as to be able to make use of Cunningham's invariant Liénard-Wiechert expression, and satisfying the wave equation;
2) to formulate a law of motion in terms of an integrand similar to Silberstein's $ds - 2\Phi ds$, but, at the same time making sure that his integrand would lead to the Einstein-prediction with regard to the precession of the perihelion of Mercury, which was shown possible by Gerber's example of calculating this precession in terms of a retarded potential of gravitation.

In fact, instead of Silberstein's integrand $ds - 2\Phi ds$, Whitehead took *m* times the square root of $dG_m^2 - \frac{2}{c^2}\sum_M \psi_M dG_M^2$, where *m* denotes the point-mass of which he wants to express the motion in terms of the Minkowskian metric dG^2 and in terms of the invariant retarded potentials of gravitation ψ_M, associated with the distribution of all other point-masses *M*, and satisfying the wave equation. This integrand at the same time harmonized with his philosophical intuitions, involved an Einstein-like tensor, and led to the correct prediction with regard to the precession of the perihelion of Mercury. In §4 I will show how Whitehead derived this integrand, which incorporated his philosophical views, as well as all he had learned from electromagnetism and relativity.

§4 Heuristic derivation of Whithead's law of gravitation

Actually, according to Whitehead, gravitational physics is not about the movement and causal influence of mass-particles in space, but about spatio-temporal and causal networks of events characterized by definite masses. One type of such networks are historical routes of events characterised by definite masses. An historical route of events characterised by a definite mass is *not* a causally impotent path, consisting of points which are successively occupied by a moving mass-particle. On the contrary, every stretch of an historical route of events characterized by a definite mass has the potential to causally influence its own historical route as well as other historical routes of events characterized by definite masses.

To fully explain the above paragraph would lead us deep into the philosophical aspects of Whitehead's **R**, which are not the topic of this lecture. However, it is essential to retain from the above paragraph the idea that, according to Whitehead, gravitation is about the causal relatedness of historical routes of mass-occurrences in space-time, which amounts to the causal relatedness of infinitesimal stretches of such historical routes. Without this idea, one can never understand how Whitehead arrived at his law of gravitation. So, once more: Whitehead's law is in essence about causally correlated infinitesimal stretches of different historical routes of mass-occurrences in the Minkowskian space-time.

Consider a particular historical route of occurrences characterised by a definite mass *m* (the *m*-route) as well as all other such historical routes, represented by an arbitrary historical route of occurrences characterised by a mass *M* (an *M*-route). Also, consider two infinitesimal intervals in the Minkowskian space-time: *XX'*, part of the *m*-route, and defined by the points $X = (x_m, y_m, z_m, t_m)$ and $X' = (x_m + dx_m, y_m + dy_m, z_m + dz_m, t_m + dt_m)$; and infinitesimal interval *PP'*, part of an *M*-route, and defined by the points $P = (x_M, y_M, z_M, t_M)$ and $P' = (x_M + dx_M, y_M + dy_M, z_M + dz_M, t_M + dt_M)$. Given the metric of the Minkowskian space-time in equation (4), the squares of the interval-lengths are:

(8) $$dG_m^2 = c^2 dt_m^2 - dx_m^2 - dy_m^2 - dz_m^2$$

(9) $$dG_M^2 = c^2 dt_M^2 - dx_M^2 - dy_M^2 - dz_M^2$$

According to Whitehead, the gravitational field expresses the joint causal influence of all historical routes of mass-occurrences, and he defines the gravitational field as an assemblage of infinitesimal elements of mass impetus. (Cf. **CN** 181) Each infinitesimal element of mass impetus is symbolised by *dI;* and each *dI* expresses the joint causal influence of all historical routes of mass-occurrences on a stretch involving an infinitesimal interval *XX'*.

The first causal influence the mass impetus *dI* has to incorporate is the causal influence on the stretch involving *XX'* exercised by its own route, the *m*-route. Whitehead defines this causal influence as the product of the mass *m* and the length *XX'*. Now suppose for a moment that this is the only causal influence to consider, then the equation expressing mass impetus dI simply is:

(10) $$dI = m\sqrt{dG_m^2}$$

This means that even if there was only one historical mass-route, Whitehead's mass impetus would not be zero. In a sense, this means that infinitesimal stretches of historical mass-routes are *causa sui* (in terms of the length factor; cf. **PR** 86 & 222) and determined by their own past (in terms of the mass-factor).

But let's not further explore the philosophy of the mass impetus *dI*. The presupposition of equation (10) is wrong. The mass impetus also has to incorporate the causal influence exercised by all *other* mass-routes on the stretch involving *XX'*. Therefore, Whitehead replaced (10) with:

(11) $$dI = m\sqrt{dJ^2}$$

dJ^2 is that which Whitehead called the potential mass impetus, and it is given by:

(12) $$dJ^2 = dG_m^2 + ...$$

This incomplete equation has to be completed by adding a term representing the potential causal influence exercised by all other *M*-routes on the stretch involving *XX'*. In fact, Whitehead did not add one term, but as much identical terms as there are other M-routes, hence making his equation linear – an aspect which Whitehead considers as a obvious advantage of his theory over Einstein's. (Cf. **R** 84)

The potential causal influence of an *M*-route on the stretch involving *XX'*, however, is limited to a particular stretch of the *M*-route, namely the stretch involving the infinitesimal interval *PP'* which is causally correlated to *XX'* as follows:

(13) $$c(t_m - t_M) = r = \sqrt{(x_m - x_M)^2 + (y_m - y_M)^2 + (z_m - z_M)^2}$$

Whitehead's phrasing for this causal correlation is: "*X* lies in the causal future of *P*." (**R** 77) It is clear from equation (13) that Whitehead holds the potential causal influence of an *M*-route to propagate at the speed of light, and that *P* is taken such that what is propagated from $\vec{x}_M = (x_M, y_M, z_M)$ can exactly reach $\vec{x}_m = (x_m, y_m, z_m)$ in time span $t_m - t_M$. Linking (13) to §3 of my paper: this equation is identical to Cunningham's *"law of propagation."* (Cf. Cunningham 1914:174)

The contribution to the potential mass impetus dJ^2 by the stretch of the *M*-route involving *PP'* is not only determined by dG_M^2, the square of the length of *PP'*, but also by a second factor involving *M,* which I denote with Ψ_M, such that equation (12) turns into:

(14) $$dJ^2 = dG_m^2 + \sum_M \Psi_M dG_M^2$$

To recapitulate: we're searching for a law of gravitation, and are now looking for a second factor Ψ_M (next to the factor dG_M^2) to express the potential causal influence, or potential mass impetus, of a stretch of an *M*-route involving *PP'* on the stretch of the *m*-route involving *XX'*. Of course, the search for a potential gravitational influence might bring to mind the classical gravitational potential ψ_M, and we might try to define Ψ_M as

(15) $$\Psi_M = \alpha\psi_M \text{ with } \psi_M = \frac{\gamma M}{r}$$

The reason why we do not identify Ψ_M and ψ_M, but take them as equal up to a constant factor α will become clear at a later stage in our derivation. Definition (15), however, will not do, because it involves a classical *instantaneous* potential, and *a fortiori* cannot be part of a relativistic theory of gravitation: the denominator *r* is not Lorentz invariant. What we are looking for is a *retarded* potential, embodying, in a Lorentz invariant fashion, the fact that gravitation propagates at the speed of light according to equation (13). The quest for such a potential, of course, leads us back again to §3, more specifically, to Cunningham's treatment of the retarded Liénard-Wiechert potentials. (Cf. Cunningham 1914:108)

The Liénard-Wiechert potentials in the Lorentz invariant format Cunningham gave them, have as denominator **V.R**, or, written in full:

(16) $$\left(1 - \frac{v_E^2}{c^2}\right)^{-\frac{1}{2}} \left(r - \frac{\vec{v}_E \cdot (\vec{x}_e - \vec{x}_E)}{c}\right)_{ret}$$

In expression (16) *e* and *E* refer to charges which play the role of m and M in our story, and the subscript *ret* stands for "retarded," which means that $r = c(t_e - t_E)$. If we turn this electromagnetic expression into a gravitational one by substituting *m* for *e*, and *M* for *E*, it gives us an appropriate Lorentz invariant generalisation of the non-relativistic *r* in definition (15). Indeed, for speeds v_E which are small compared to *c,* this expression simply reduces to *r*. Replacing *r* with the gravitational analogue of expression (16) in definition (15) yields:

$$(17)\quad \Psi_M = \alpha\psi_M \text{ with } \psi_M = \frac{\gamma M}{\left(1-\frac{v_M^2}{c^2}\right)^{\frac{-1}{2}}\left(r-\frac{\vec{v}_M\cdot(\vec{x}_m-\vec{x}_M)}{c}\right)_{ret}}$$

Substituting definition (17) for Ψ_M in equation (14) for the potential mass impetus gives:

$$(18)\quad dJ^2 = dG_m^2 + \alpha\sum_M \psi_M dG_M^2 \text{ with } \psi_M = \frac{\gamma M}{\left(1-\frac{v_M^2}{c^2}\right)^{\frac{-1}{2}}\left(r-\frac{\vec{v}_M\cdot(\vec{x}_m-\vec{x}_M)}{c}\right)_{ret}}$$

Equation (18) is almost Whitehead's law of gravitation [equation (5)], where ψ_M is not the classical Newtonian potential of gravitation, but a relativistic retarded potential of gravitation conforming Cunninghams law of propagation. Only one task is left: to show that the as yet unspecified constant α can best be taken as equal to $-2/c^2$. The best way to do so, is to submit Whitehead's not yet fully specified law of gravitation in equation (18) to the requirement that the law of motion it entails approximates Newton's law of motion.

We have already seen that the relativistic retarded potential of gravitation ψ_M in equation (18) turns into the classical Newtonian potential of gravitation $\psi_M = \frac{\gamma M}{r}$ for speeds v_M which are small compared to *c*. But what about dG_m^2 and dG_M^2 ? In general, the former is given by equation (8), and the latter by equation (9). By respectively putting dt_m^2 and dt_M^2 upfront in the left hand sides of these equations, they become:

$$(19)\quad dG_m^2 = dt_m^2\left(c^2 - v_m^2\right)$$

$$(20)\quad dG_M^2 = dt_M^2\left(c^2 - v_M^2\right)$$

In order to specify α, we limit ourselves to the particular case of a mass *m* moving slowly at speed $\sqrt{\dot{x}^2+\dot{y}^2+\dot{z}^2}$ compared to a large mass *M,* which is taken to be at rest (its centre is taken as the origin of the reference frame), and hence, of which the speed is 0. In this classical case, we do not have to worry about retardation, and hence $t_m = t_M = t$. Consequently, equations (19) and (20) become:

$$(21)\quad dG_m^2 = dt^2\left(c^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2\right)$$

(22) $dG_M^2 = dt^2 c^2$

Substituting the Newtonian potential of gravitation, $r = \sqrt{x^2 + y^2 + z^2}$, dG_m^2 and dG_M^2 in equation (18) yields:

(23) $dJ^2 = dt^2\left(c^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2\right) + \alpha\gamma M\left(x^2 + y^2 + z^2\right)^{\frac{-1}{2}} dt^2 c^2$

Putting dt^2 at the end of the left hand side, taking the square root of both sides and then multiplying both with *m* yields:

(24) $m\sqrt{dJ^2} = m\left(c^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 + \alpha c^2\gamma M\left(x^2 + y^2 + z^2\right)^{\frac{-1}{2}}\right)^{\frac{1}{2}} dt$

Given the fact that $m\sqrt{dJ^2}$ is the integrand of Whitehead's law of motion (6) for any *m*, equation (24) implies for our particular case that the motion of *m* is governed by:

(25) $\delta\int L dt = 0$ with $L = m\left(c^2 - \dot{x}^2 - \dot{y}^2 - \dot{z}^2 + \alpha c^2\gamma M\left(x^2 + y^2 + z^2\right)^{\frac{-1}{2}}\right)^{\frac{1}{2}}$

From the calculus of variations, we know that solving equation (25) amounts to solving the Euler-Lagrange equations:

(26) $\frac{d}{dt}\frac{\partial L}{\partial \dot{x}} - \frac{\partial L}{\partial x} = 0$ and $\frac{d}{dt}\frac{\partial L}{\partial \dot{y}} - \frac{\partial L}{\partial y} = 0$ and $\frac{d}{dt}\frac{\partial L}{\partial \dot{z}} - \frac{\partial L}{\partial z} = 0$

Substituting the long expression for *L* in these equations, carefully calculating the differentials, dividing away the appropriate non-zero factors, and re-substituting *r* at the end of the calculation, they become:

(27) $m\frac{d\dot{x}}{dt} - \frac{\alpha c^2\gamma m M x}{2r^3} = 0$ and $m\frac{d\dot{y}}{dt} - \frac{\alpha c^2\gamma m M y}{2r^3} = 0$ and $m\frac{d\dot{z}}{dt} - \frac{\alpha c^2\gamma m M z}{2r^3} = 0$

We can summarize this result in one vector equation by introducing the acceleration vector $\vec{a}$ and the unit vector pointing from *m* to *M*, or $\vec{e} = -\vec{x}/r$:

(28) $m\vec{a} = \frac{-\alpha c^2\gamma m M}{2r^2}\vec{e}$

Only if we put $\alpha = -2/c^2$ this yields Newton's law of motion in the case at hand, or:

(29) $m\vec{a} = \frac{\gamma m M}{r^2}\vec{e}$

Consequently, we do in fact put $\alpha = -2/c^2$ and equation (18) turns into:

(30) $$dJ^2 = dG_m^2 - \frac{2}{c^2}\sum_M \psi_M dG_M^2 \text{ with } \psi_M = \frac{\gamma M}{\left(1-\frac{v_M^2}{c^2}\right)^{\frac{-1}{2}}\left(r - \frac{\vec{v}_M \cdot (\vec{x}_m - \vec{x}_M)}{c}\right)_{ret}}$$

Equation (30) is identical to equation (5), which is Whitehead's law of gravitation. Given the fact that (30) is written in terms of invariants, and that the particular case to determine the constant factor already represents the case of approximation to Newton's theory, I rush to the conclusion that Whitehead's law of gravitation indeed satisfies the first two of his requirements: (i) to be invariant, and (ii) to approximate Newton. That his law of gravitation also satisfies requirement (iii) – to be empirically equivalent with Einstein – is the topic of §5 of this paper.

Whitehead added an important remark after formulating his law of gravitation in **R**: "in an empty region ψ_M satisfies $\nabla^2\psi - \frac{1}{c^2}\frac{\partial^2\psi}{\partial t^2} = 0$. We might have started from [this] differential equation as the only invariant form of linear differential equation of the second order, and then deduced the above solution for ψ_M as the only invariant solution for a single point-wise discontinuity. The procedure of thought which I have adopted seems to me to be better suited to throw into relief the fundamental ideas concerning nature." (**R** 82) Whitehead's remark on the wave equation, and on its solution for a point-charge or a point-mass, comes as no surprise after highlighting the Silberstein-Cunningham context. It just confirms that his work was in fact rooted in the physics of the people surrounding him.

Furthermore, I agree with Whitehead that it is important to throw into relief the fundamental ideas concerning nature. Indeed, scientific concepts "are not to be looked for at the tail end of a welter of differential equations." (**PNK** vi) And I agree with C. D. Broad, the author of the critical April 1923 *Mind* review of **R**, that Whitehead's "law of gravitation appears as an equation involving infinitesimal stretches along historical routes," and that "this is perhaps the most original and philosophically interesting feature of his theory." (p.219) But I also think that Whitehead's neglect to link his theory to the pioneering research of his colleagues, has obscured the fact that his theory was not only of philosophical interest and *ad hoc,* but was also rooted in the work of other applied mathematicians and physicists. This neglect is one of the reasons why his work did not have a significant impact on the further course of physics, even though Whitehead's 1922 theory accounted for all the GTR-phenomena Einstein accounted for or predicted, *including* the 43" per century extra rotation of the orbit of Mercury.

Notice that Whitehead's remark on the necessity to throw into relief the fundamental ideas concerning nature is a direct critique of Einstein, who did introduce his law of gravitation by mathematical analogy to another differential equation. E.g., in §16 of his 1916 article on "The Foundation of the General Theory of Relativity," Einstein introduced "The General Form of the Field Equations of Gravitation" by stating: "The field equations for matter-free space formulated in §15 are to be compared with the field equation $\nabla^2\phi = 0$ of Newton's theory. We require the equation corresponding to Poisson's equation $\nabla^2\phi = 4\pi\gamma\rho$ where ρ denotes the density of matter." (Einstein 1916:148) For Whitehead, mathematical analogies may not darken the fundamental concepts, for Einstein, "Poisson's equation […] of the Newtonian theory must serve as a model." (Einstein 1922:82)

This being said, it is clear that the same classical equations – Poisson's and Laplace's, d'Alembert's and the wave equation (the latter two can also be called the inhomogeneous and the homogeneous wave equation) – form part of the background of Whitehead's discovery of his law of gravitation. As said, Whitehead's law is closely correlated to the wave equation for point-masses, which is similar to the wave equation for point-charges, of which the solutions define the Liénard-Wiechert potentials of electrodynamics. On the other hand, Einstein's law is a kind of generalisation of Poisson's equation for mass density. E.g., Einstein's formulation of "Newton's Theory as a First Approximation" yields Poisson's equation. (Einstein 1916:159)

Einstein's law is closely related to that other generalisation of Poisson's equation, d'Alemberts equation. This relationship is clearly revealed in the case of weak gravitational fields (cf., e.g., Rindler 1979:188-192): then Einstein's g_{ik} can be written as $\eta_{ik} + h_{ik}$, where η_{ik} is the Minkowski tensor and h_{ik} the small difference tensor, and Einstein's law can be replaced by the linear approximation

$$(31) \qquad \nabla^2 h_{ik} - \frac{1}{c^2}\frac{\partial^2 h_{ik}}{\partial t^2} = 2\kappa\left(T_{ik} - \frac{1}{2}\eta_{ik}T\right)$$

where $T = \kappa R$, and the last factor of the right hand side clearly is the mass-energy density factor. Einstein noticed with regard to (31) that "these equations may be solved by the method, familiar in electrodynamics, of retarded potentials." (Einstein 1922:87) Silberstein noticed: "The field equations […] now become d'Alembert's equations." (Silberstein 1924:388; see also 430-434) Wolfgang Rindler's comment on (31) reads: "Disturbances of the field will therefore be propagated through vacuum at the speed of light. […] among these [are] genuine gravitational waves." (Rindler 1979:190) When solving (31) Rindler notices:

"The formal similarity with Maxwell's theory is striking." (Rindler 1979:191) No surprise, for in the case of charge density, the solutions of d'Alembert's equation define the general solutions of Maxwell's field equations.

The difference between Whitehead's law and Einstein's law is clearly related to the difference between the wave equation and d'Alembert's equation. I guess, that is why this difference has been described by J. L. Singe in 1956 "as analogous to the difference of the Liénard-Wiechert retarded potentials particle formulation of classical electrodynamics and the Maxwell field formulation." (Bain 1998:footnote 26) However, I hold the latter statement to be ambiguous. If it means that the Liénard-Wiechert potentials exclude the field-concept because they are about particles (charge-points/mass-points), and if it implies that Whitehead's theory is a retarded *action-at-a-distance* theory, it is simply wrong. As Cunningham's treatment of the Liénard-Wiechert potentials clearly shows – and so does Feynman's treatment in *The Feynman Lectures on Physics, Volume II*, §21.5 – the difference is not between a non-field theory and a field theory, but between discrete charges/masses and continuous charge/mass densities. In both charge-cases, the electromagnetic vector fields $\vec{E}$ and $\vec{B}$ can be expressed in terms of the vector and scalar potentials $\vec{A}$ and ϕ. In both Whitehead's and Einstein's mass-theories, the gravitational field can be expressed in terms of a field tensor, respectively, Whitehead's potential mass impetus tensor J_{ik}, and Einstein's fundamental tensor g_{ik}.

This brings us smoothly to §5, in which I will explore the empirical equivalence of Whitehead's and Einstein's theories in terms of their respective tensors J_{ik} and g_{ik}, or, which amounts to the same, in terms of their respective line-elements, given by means of $dJ^2 = J_{ik} dx^i dx^k$ and $ds^2 = g_{ik} dx^i dx^k$.

§5 Schwarzschild equivalence

The first and most important exact solution of Einstein's equation (1) was found in 1916 by Karl Schwarzschild. For a spherically symmetric mass *M* (such as the sun) the fundamental tensor g_{ik} – embodying (according to Einstein) both the metric and the gravitational field – is given by:

$$(32)\quad \begin{pmatrix} -\left(1-\frac{2M}{r}\right)^{-1} & 0 & 0 & 0 \\ 0 & -r^2 & 0 & 0 \\ 0 & 0 & -r^2\sin^2\theta & 0 \\ 0 & 0 & 0 & 1-\frac{2M}{r} \end{pmatrix}$$

where g_{ik} is expressed in the spherical coordinates r, θ, ϕ, t of a coordinate frame with the centre of *M* as its origin. The importance of Schwarzschild's solution lies in the fact that after substituting (32) in Einstein's line-element (2), his laws of motion (3) easily yield the light-bending, perihelion-precession and red-shift predictions.

Let's see what Whitehead's gravitational field looks like for a spherically symmetric mass *M* and a spherically coordinate frame as above (implying $\vec{v}_M = 0$). To that end, consider a test mass *m* ($\vec{v}_m$ not necessarily small) causally correlated with *M*. Then:

$$(33)\quad dG_m^2 = c^2dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2$$

$$(34)\quad dG_M^2 = c^2dt_M^2 = (cdt - dr)^2 \text{ because } c(t - t_M) = r$$

$$(35)\quad \psi_M = \gamma M/r$$

Substituting (33), (34) and (35) in equation (30), and choosing – without loss of generality – *c* and γ as units, yields:

$$(36)\quad dJ^2 = dt^2 - dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 - \frac{2M}{r}(dt - dr)^2$$

In a 1924 letter to the editor of *Nature*, entitled "Comparison of Whitehead's and Einstein's Formulae," Arthur Eddington highlighted that by introducing a new coordinate t_1, given by:

$$(37)\quad t_1 = t + 2M\log(r - M)$$

Whitehead's equation (36) turns into:

$$(38)\quad dJ^2 = -\left(1-\frac{2M}{r}\right)^{-1}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\phi^2 + \left(1-\frac{2M}{r}\right)dt_1^2$$

The calculation is straightforward: differentiation of (37) implies

$$(39)\quad dt = dt_1 - \frac{2M}{r-M}dr$$

and substitution of (39) in (36) – after some manipulation – leads to (38).

As $dJ^2 = J_{ik}dx^idx^k$, (38) means that the tensor J_{ik} – embodying (according to Whitehead) the gravitational field – is also given by (32). In other words, in the case of a spherically symmetric mass *M*, the Schwarzschild solution of Einstein's equation (1) is also an exact solution of Whitehead's equation (5). But the identity of g_{ik} and J_{ik} leads to the identity of ds^2 and dJ^2, and *a fortiori*, of the laws of motion which yield the three famous Einstein-predictions. Consequently, Whitehead's theory also satisfies his requirement (iii), and Whitehead's conclusion (upon comparing Einstein's formula with his) is appropriate: "So far as matters stand at present both formulae give the motion of Mercury's perihelion, […] a possible shift of the spectral lines […], and lastly, […] the famous eclipse results […]." (**R** 84) However, the quoted conclusion is a 1922 state of the art conclusion. Today "it is not sufficient to check the 'classic tests' […]. There is now an exhaustive battery of empirical checks that must be done." (Gibbons & Will 2006:5)

True, Whitehead's and Einstein's theory share the Schwarzschild solution and hence give the same predictions for the original empirical tests which played a crucial role in the history of GTR. "In fact an even stronger statement can be made. This remarkable correspondence of exact solutions extends to the Kerr solution (Russell & Wasserman 1987) and thus to the corresponding […] frame dragging effects. Thus experiments such as […] the ongoing NASA-Stanford Gravity Probe B superconducting gyroscope experiment […] cannot distinguish Whitehead's from Einstein's theory on the basis of frame dragging […]." (Gibbons & Will 2006:2) However, the empirical equivalence of Whitehead's and Einstein's theory of gravitation does not necessarily hold for test-cases which go beyond the presuppositions leading to the particular Schwarzschild and Kerr solutions they share. In fact, some of today's refined experiments, involving the solar system, binary pulsars, etc., can no longer be described in terms of the line-elements which can be derived from both Einstein's and Whitehead's theory. Hence, in their 2006 article with the telling title "On the Multiple Deaths of Whitehead's Theory of Gravity," Gary Gibbons and Clifford Will argue "that Whitehead's theory is definitely excluded by several modern experiments, and [that] any one of them is sufficient for rejection." (p.4) Gibbons and Will discuss five specific tests which Whithead's theory fails to pass, and they conclude: "In other words, judged by the modern scientific and technological standards, Whitehead's theory, beautiful as it may seem in the eyes of many of its beholders, is truly dead. By contrast, Einstein's theory passes all of these tests with flying colours." (p.4)

Even though other admirers of Whitehead might disagree, I will not look for weak points in the article of Gibbons and Will. I simply accept their expertise and their overall conclusion. My reason for doing so is what Whitehead wrote in this context on the topic of experimental observation: “If the above formula [(30)] gives results which are discrepant with observation, it would be quite possible with my general theory of nature to adopt Einstein’s formula, based upon his differential equations, for the determination of the gravitational field. […] Perhaps neither of the above formulae [his and Einstein’s, RD] will survive further tests of other delicate observations. In this event we are not at the end of our resources. There are, in addition to Einstein’s, yet two other sets of tensor differential equations which on the theory of nature explained in this lecture satisfy all the general requirements.” (**R** 84)

What Whitehead proposed in case of experimental failure of his law of gravitation, and success of Einstein’s law, is to drop the first and to adopt the latter *for the determination of the gravitational field;* not for the determination of the metric of space-time – of course, for this would be a violation of Whitehead’s general theory of nature. Furthermore, what he proposed in case of experimental failure of both his and Einstein’s theory, is to examine the two other alternatives he indicated in his 1922 book. In §6, taking for granted Gibbons and Will’s claim that Einstein’s theory passes all tests at hand with flying colours, I will focus on the possibility to utilize Einstein’s law of gravitation for the determination of the gravitational field in the context of Whitehead’s philosophy of nature.

§6 Rosen, Gupta and Feynman

In 1960 Robert Palter published a book on *Whitehead’s Philosophy of Science* which I recommend to all readers interested in Whitehead’s theory of relativity. It includes Palter’s comment on Whitehead’s statement about the possibility to adopt Einstein’s formula. Palter wrote**:** “Whitehead is willing to accept Einstein’s mathematical formulae for the laws of motion and gravitation (provided, of course, these formulae are reinterpreted in terms of Whitehead’s own natural philosophy). […] It is not perfectly plain just how one should interpret Einstein’s general theory of relativity in such a way as to be consonant with Whitehead’s natural philosophy. Unfortunately, Whitehead does not elaborate on this point and we are compelled to guess at his meaning. […] I believe Whitehead’s reinterpretation of Einstein’s general theory of relativity must involve the splitting up of Einstein’s fundamental tensor, g_{ik}, into two parts, the one consisting of a tensor with the constant components,

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix}$$

representing purely spatio-temporal or intertial properties; and the other consisting of a tensor with variable components representing the gravitational field." (pp.207-208)

As support for the possibility of the split of Einstein's fundamental tensor in a spatio-temporal and a gravitational part, Palter referred to Arthur Eddington's 1923 *The Mathematical Theory of Relativity*. Indeed, Eddington wrote: "The field described by the g_{ik} may be (artificially) divided into a *field of pure inertia* represented by the Galilean values, and a *field of force* represented by the deviations of the g_{ik} from the Galilean values." (p.95) The similarity with Silberstein's 1918 approach, which may have inspired Whitehead to start with (cf. §3), is obvious. Only, neither Eddington, nor Palter, developed this idea into a full theory. And yet, if Palter had carefully read the 1953 second volume of Edmund Whittaker's *A History of the Theories of Aether and Electricity*, he might have noticed that Whitttaker already hinted at such a theory.

The second volume of *A History of the Theories of Aether and Electricity* is notorious among historians of science for maximising Poincaré's contribution to STR, and for minimising Einstein's. In this context, however, the important part is its short description of Whitehead's theory of gravitation (on pp.174-175). Whittaker, once Whitehead's student at Cambridge, and his friend ever after, concluded his description with the following remark: "Whitehead's doctrine, though completely different from Einstein's in its formulation, may be described very loosely as fitting the Einsteinian laws into a flat space-time […]. The idea of mapping the curved space of General Relativity on a flat space, and making the latter fundamental, was revived many years after Whitehead by N. Rosen." (p.175) Nowadays, Nathan Rosen's name is mainly remembered for his joint 1935 paper with Einstein and Boris Podolski (the so-called EPR paper) on the question: "Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?" Whittaker, however, refers to Rosen's 1940 *Physical Review* articles, "General Relativity and Flat Space I & II."

These articles by Rosen might have provided Palter with the kind of theory he was looking for. In line with Eddington's suggestion to *artificially* split up the field described by the fundamental tensor in an inertial field and a gravitational field, Rosen suggested to introduce a *fictional* metric γ_{ik} which imposes flatness in each point of space-time in addition to a *gravitational* metric g_{ik}. Rosen wrote: "From the standpoint of the general theory of relativity, one must look upon γ_{ik} as a fiction introduced for mathematical convenience. However, the question arises, whether it may not be possible to adopt a different point of view, one in which the metric tensor γ_{ik} is given a real physical significance as describing the geometrical properties of space, which is therefore taken to be flat, whereas the gravitational tensor g_{ik} is to be regarded as describing the gravitational field." (Rosen 1940:150) It is at once evident why Rosen's theory has been called a *bimetric* GTR (even though *bitensorial* would have been an even better name), and why it has been interpreted as a possibility to adopt Whitehead's point of view within physics.

Whereas Palter, despite the fact that Whittaker's book is mentioned in his bibliography, did not link Whitehead's theory of gravity to Rosen's bimetric approach, it has recently been done, and without reference to Whittaker, in an interesting monograph, *Whitehead and the Measurement Problem of Cosmology*. Its author, Gary Herstein, is well aware of the fact that Rosen's bimetric GTR "ran into trouble in its predictions regarding gravitational waves and […] pulsars, and [in] its failure to predict black holes," but he draws attention to the fact that "modifications of Rosen's work by Piso, Ionescu-Pallas, and Onofrei brought that theory back into a viable condition (Piso *et al* 1994), and [that] on this account one can still legitimately talk about Rosen's work as a live option in physics." (p.176) If one of the recent bimetrical theories is empirically equivalent or superior to Einstein's theory, then – according to Herstein – it can be considered as a good candidate to replace the geometry-physics separating theory which Whitehead launched in 1922, but which, meanwhile, has been falsified by Clifford Will and Gary Gibbons.

Consequently, Herstein writes: "The particular theory that Whitehead presents as an *exempli gratia* is itself representative of an entire class of such theories, many of which can equally well solve the problem Whitehead set out to deal with." (p.23) More explicitly, Herstein writes: "Clifford Will has presented evidence that claims to prove that Whitehead's theory of relativity is empirically false. [But] there are viable theories of gravity that are every bit as empirically robust as the Einsteinian GTR. In particular, there are what are known as 'bimetric' theories of gravity, so-called because they separate the purely geometric relations from the physical ones. This is what Whitehead argued was a necessary condition for a coherent theory [and] Whitehead's own theory of gravity qualifies as bimetric. It is this quality of being bimetric that makes a theory of gravity Whiteheadian. Thus the particular viability of Whitehead's theory is a matter of comparatively little consequence. There are other bimetric theories that are clearly viable, and as such Whitehead's general approach to a theory of gravity remains a live option." (p.27)

Moreover, according to Herstein, these viable bimetric theories, are more promising theories of gravitation than Einstein's to be reconciled with the quantum field theories describing the other forces of nature. And Herstein approvingly quotes Rosen, who wrote: "In the Einstein general relativity theory gravitation is explained in terms of geometry. In the theory suggested here […] this geometrization of gravitation has been given up. Perhaps this may be regarded by some as a step backward. It should be noted, however, that the geometrization referred to has never been extended satisfactorily to other branches of physics, so that gravitation is treated differently from other phenomena. It is therefore not unreasonable to wonder whether it may not be better to give up the geometrical approach to gravitation for the sake of obtaining a more uniform treatment for all the various fields of force that are to be found in nature." (Rosen 1940:150)

One might question whether the modified Rosen bimetric gravity theory (Piso *et al* 1994), or the other recent bimetric theory Herstein refers to (Moffat 2003), pass Gibbons and Will's five tests (which Einstein's theory passes, but Whitehead's theory fails to pass). But, no matter the outcome of that question, I agree with Herstein that the bimetric theories to look for, should be theories explicitly giving up the geometrical interpretation of the gravitational metric, and providing a uniform treatment of all fields of forces. Are there such theories which – on top of their Whiteheadian characteristics – are also empirically equivalent to Einstein's theory of gravitation? I think there are. But in order to introduce those theories, I first return to Robert Palter, who – four years after writing his *Whitehead's Philosophy of Science* – wrote a contribution to the Hartshorne festschrift *Process and Divinity.* In his festschrift essay, Palter again stressed "that Whitehead's attitude toward Einsteins general relativity is not in fact purely negative," and he added:

"Whitehead is quite willing, he says, to adopt Einstein's law of gravitation if it turns out to be in better agreement with observations than his own law; furthermore, Whitehead claims that his natural philosophy is not incompatible with Einstein's law. How can this be? The non-uniform geometry of general relativity would seem to be irreconcilable with Whitehead's insistence on uniform space-time. Obviously, Einstein's law must be somehow reinterpreted, but Whitehead never explains just how. Perhaps, however, a recent reformulation of Einstein's law in *flat* space-time illustrates what Whitehead had in mind. S. Gupta has succeeded in constructing a law of gravitation equivalent mathematically to Einstein's law but involving only the pseudo-Euclidean space-time of special relativity (and of Whitehead's own theory of gravitation)." (Palter 1964:67-68)

The Suraj Gupta article Palter referred to, is called “Einstein’s and Other Theories of Gravitation,” and it was published in *Reviews of Modern Physics* in 1957. In his article, Gupta first considered two of the previous “attempts to construct a theory of the gravitational field in flat space similar to the theories of the electromagnetic field and the meson fields.” (p.334) “Because the observed properties of the gravitational field are quite different from those of the electromagnetic field,” Gupta excluded a vector (spin-1) gravitational field theory (the electromagnetic field is a vector field, the electromagnetic force is mediated by the exchange of spin-1 photons), but discussed the 1914 scalar (spin-0) gravitational field theory of G. Nordström, and the 1943 tensor (spin-2) gravitational field theory of G. D. Birkhoff. Gupta rejected the first as being “definitely contrary to experiments” (p.334), and the latter because, “as pointed out by H. Weyl, this theory suffers from serious difficulties.” (p.335) But then he proceeded to show that Einstein’s own theory can be treated as a tensor spin-2 gravitational field theory in the space-time characterized by the Minkowskian metric (the gravitational field being mediated by the exchange of postulated spin 2-gravitons).

I will not enter into the details of Gupta’s reinterpretation of Einstein’s theory, but end my discussion of his article by quoting its conclusion: “We can treat Einstein’s theory as a theory of gravitation in flat space. Such a treatment has two great advantages. Firstly, it provides us with a more uniform description of the gravitational and the electromagnetic fields. Secondly, it enables us to carry out the quantization of Einstein’s gravitational field by following the same procedure as we use for the electromagnetic field […]. On quantization, Einstein’s gravitational field corresponds to gravitational quanta or gravitons of vanishing rest-mass and spin 2, and it is possible to calculate the interactions of these gravitons and other particles in the usual way. In such a quantized theory the nonlinearity of the gravitational field appears as a direct interaction between gravitons.” (p.336)

It’s clear that Gupta’s reinterpretation of Einstein’s GTR aims at a uniform treatment of all fields of force against the background of Minkowski’s space-time. To stress that this is a Whiteheadian approach, I need to add an aspect of Whitehead’s theory of gravitation which I have neglected so far. I have highlighted the similarity between the retarded potential of gravitation and the retarded potential of electromagnetism, but I have only spoken about Whitehead’s mass impetus $m\sqrt{dJ^2}$, and not about his electromagnetic impetus. However, in **R**, Whitehead presented a uniform treatment of the gravitational and the electromagnetic field, and he defined impetus as the sum of mass impetus and electromagnetic impetus. Hence equation (11) should be replaced by:

(40) $dI = m\sqrt{dJ^2} + c^{-1}EdF$

and the first law of motion in (6) should be replaced by:

(41) $\delta\int dI = 0$

In the mass impetus *m* is a mass and *J* is a tensor (a symmetric covariant tensor of the second order); in the electromagnetic impetus *E* is a charge and *F* is a vector (a covariant tensor of the first order). So one might say that Whitehead's approach is a forerunner of Gupta's approach, or reversing it, that Gupta's approach is Whiteheadian, not only in its use of the Minkowskian metric, but also in its uniform treatment of the known fields of forces.

In fact, one of the aspects of Einstein's theory, an aspect Whitehead always considered as of major importance, is that it necessitates a change in the theory of electromagnetism. Whitehead wrote: "It is an outcome of Einstein's work that the electro-magnetic equations require modification to express the association of the gravitational and electro-magnetic fields. This is one of his greatest discoveries. The most natural deduction to make from these modified equations is that the velocity of light is modified by the gravitational properties of the field through which it passes." (**ESP** 334) Or again: "The electromagnetic theory has to be modified to allow for the presence of a gravitational field. Thus Einstein's investigations lead to the first discovery of any relation between gravity and other physical phenomena." (**CN** 183) Consequently: "In the absence of gravitational fields, Whitehead adopts the standard Maxwell-Lorentz field equations for electromagnetic phenomena. However, in the presence of a gravitational field, Whitehead modifies the Maxwell-Lorentz equations to take account of the influence of gravitation on electromagnetic phenomena." (Palter 1960:204) Anyway, it's his awareness of the interrelatedness of all fields of force which prompted Whitehead to aim at a unified treatment of gravitation and electromagnetism.

Of course, in Whitehead's days, only the gravitational and electromagnetic fields had to be captured in a uniform treatment, whereas in Gupta's days other fields of force had entered the arena of physics. This brings us at once to the *Lectures on Gravitation* Richard Feynman gave at Caltech during the academic year 1962-63. Feynman's "Field Approach to Gravitation" (§1.1) starts from the observation that in Einstein's days, physics "consisted simply in gravitation and electrodynamics, and electrodynamics had forced the invention of a theory of relativity, so that the problem was to bring the theory of gravitation into line with the discoveries that had been made by studying electrodynamics." According to Feynman, Einstein's GTR "resulted in beautiful relations connecting gravitational phenomena with the geometry of space", and this exciting idea made every one dream "he would find the way of geometrizing electrodynamics." However, despite the fact that "a generation of physicists worked trying to make a so-called unified field theory," none of them has been successful. Moreover, "such a success would have been short lived," because "there is so much more in the world besides electricity and gravitation."

This brought Feynman to his own "pedagogical approach," which is more suited to quantum field theorists who have gotten used to the idea of a multiplicity of fields, and who might consider the gravitational field as just "another such field." Hence Feynman described his approach in terms of a fiction: "Imagine that in some small region of the universe, say a planet such as Venus, we have scientists who know all about the other […] fields of the universe […], but who do not know about gravitation. And suddenly, an amazing new experiment is performed, which shows that two large neutral masses attract each other with a very, very tiny force. Now, what would the Venutians do with such an amazing extra experimental fact to be explanatory? They would probably try to interpret it in terms of the field theories which are familiar to them." Based on his pedagogical field approach, Feynman succeeded in recreating GTR from a non-geometrical viewpoint. His approach "develops the theory of a massless spin-2 field (the graviton) coupled to the energy-momentum tensor of matter, and demonstrates that the effort to make the theory self-consistent leads inevitably to Einstein's general relativity." (Preskill & Thorne 1999:ix)

This is not the place to enter into the details of Feynman's *Lectures on Gravitation*, but let's give a feel of them by quoting the "Foreword" of John Preskill and Kip S. Thorne: "With hindsight, we can arrive at Maxwell's classical electrodynamics by starting with the observation that the photon is a massless spin-1 particle. The form of the quantum theory of a massless spin-1 particle coupled to charged matter is highly constrained by fundamental principles such as Lorentz invariance and conservation of probability. The self-consistent version of the quantum theory – quantum electrodynamics – is governed, in the classical limit, by Maxwell's classical field equations. Emboldened by this analogy, Feynman views the quantum theory of gravitation as 'just another quantum field theory' like quantum electrodynamics. Thus he asks in Lectures 1-6: can we find a sensible quantum field theory describing massless spin-2 quanta (gravitons) coupled to matter, in ordinary flat Minkowski spacetime? The classical limit of such a quantum theory should be governed by Einstein's general relativistic field equation for the classical gravitational field. Therefore, to ascertain the form of the classical theory, Feynman appeals to the features of the quantum theory that must underlie it. Geometrical ideas enter Feynman's discussion only through "the back door," and are developed primarily as technical tools to assist with the task of constructing an acceptable theory." (Preskill & Thorne 1999:x-xi)

Is it too farfetched to say that Feynman's approach is Whiteheadian? Whitehead defined a Lorentz invariant field-impetus based on charges to arrive at an electrodynamics of which the equations are generalizations of the Lorentz-Maxwell equations; and he used an identical approach for gravitation: he defined a gravitational, Lorentz invariant field-impetus based on masses to arrive at his law of gravitation, yielding (or so he hoped) the same (or better) predictions as Einstein's equation. Feynman defined the Lorentz invariant field of massless spin-1 particles (photons) coupled to charged matter to arrive at quantum electrodynamics, which – in the classical limit – amounts to Maxwell's equations; and he used an identical approach for gravitation: he defined the Lorentz invariant field of massless spin-2 particles (gravitons) coupled to the energy-momentum tensor of matter to arrive at a quantum field theory of gravitation, yielding – in the classical limit – Einstein's equation of gravitation.

Moreover, Whitehead wanted "to maintain the old division between physics and geometry." (**R** v) So despite his adoption of "Einstein's method of using the theory of tensors" (**CN** vii), he avoided its geometrical interpretation, and wrote: "The theory of tensors is usually expounded under the guise of geometrical metaphors which entirely mask the type of application which I give [in **R**]. For example, the whole idea of any 'fundamental tensor' is foreign to my purpose and impedes the comprehension of my applications." (**R** vi) Similarly, for Feynman, according to Preskill and Thorne, "the conventional geometrical approach to gravitation obscures the telling analogy between gravitation and electrodynamics." Brian Hatfield adds: "Feynmann certainly felt that the geometrical interpretation is 'marvellous' (§8.2) but the fact that a massless spin-2 field can be interpreted as a metric was simply a 'coincidence' that 'might be understood as representing some kind of gauge invariance.'" (Hatfield 1999:xxxii) And David Kaiser writes that Feynman's "approach was entirely un-geometrical. The theory was based on the assumption that 'space is describable as the space of Special Relativity' (Feynman 1999:112). The tensor g_{ik}, rather than serving as a metric tensor to define a curved Riemannian manifold, was simply a convenient combination of the Kronecker delta, δ_{ik}, [or better, of the Minkowski metric, η_{ik}, RD] and the fundamental gravitational field, h_{ik}. […] Feynman had thus made a complete break with the geometrical conception of gravitation: just like the exchange of photons in quantum electrodynamics, gravitation was due to the exchange of spin-2 gravitons. There was no need to interpret the various symbols […] as anything more than the necessary algebraic apparatus of a non-linear field theory. Feynman concluded rather succinctly: 'The geometric interpretation is not really necessary or essential to physics' (Feynman 1999:113)." (Kaiser 1998:330-331)

Even without touching a third similarity – both Whitehead (cf. **SMW** 62 & 106) and Feynman insisted on the fact "that our equations be deducible from a variational principle such as Least Action" (Feynman 1999:75) – I think the above justifies calling Feynman's approach Whiteheadian. Of course, my point is not to invoke Feynman's authority. I will end §6 with the remark that, ultimately, Feynman did not establish a satisfactory theory of gravitation. Also, it is possible to defend Einstein's approach against Feynman's, as is shown, e.g., in Roberto Torretti's "Gravity as Spacetime Curvature." [Cf. Torretti 2000:129-131; notice that Torretti also defends Einstein's geometrical approach versus Harold Jeffreys' a-geometrical reading of GTR (pp.128-129), and that Jeffreys was close to Whitehead; e.g., next to Whitehead and Eddington, Jeffreys was present at Lord Haldane's London home for a meeting with Einstein in 1921, and G. Temple explicitly mentions Jeffreys article on "The Relation between Geometry and Einstein's Theory of Gravitation" (written together with Dorothy Wrinch) as part of the background against which to understand Silberstein's, Whitehead's, and his own 1923 approach.] However, like Palter, who noticed that Gupta's approach might be a way of reconciling Whitehead's philosophy of nature with Einstein's GTR, I think it is valuable to notice that Feynman's approach might also represent such a way.

Actually, Gupta's and Feynman's approach are two of a kind, and had a precedent in Robert Kraichnan's research. Preskill and Thorne write: "The field equation for a free massless spin-2 field was written down by Fierz and Pauli in 1939. Thereafter, the idea of treating Einstein gravity as a theory of a spin-2 field in flat space surfaced occasionally in the literature. As far as we know, however, the first published attempt to *derive* the nonlinear couplings in Einstein's theory in this framework appeared in a 1954 paper by Suraj N. Gupta. […] Some years before Gupta's work, Robert Kraichnan, then an 18-year old undergraduate at M.I.T., had also studied the problem [and] described his results in his unpublished 1946-47 Bachelor's thesis. […] Kraichnan did not publish any of his results until 1955 […]. It seems likely that Feynman was completely unaware of the work of Gupta and Kraichnan." (Preskill & Thorne 1999:xiii-xiv)

Of course, with the theories of Kraichnan, Gupta, and Feynman, the history of quantum gravity did not come to an end, and it is necessary to briefly characterise the three stages in the history of quantum gravity which Abhay Ashtekar – one of the fathers of loop quantum gravity, next to Carlo Rovelli and Lee Smolin – differentiated in his paper "The Winding Road to Quantum Gravity."

"First, there was the beginning: exploration. The goal was to do unto gravity as one would do unto any other physical field." (Ashtekar 1991:2) It is clear that the explorations of Kraichnan, Gupta, and Feynman, belong to this first stage, and what Ashtekar writes on the conceptual problems of Einstein's GTR which a field-theoretic approach might solve, points exactly at the problems Whitehead saw and tried to solve in 1922 – hence, confirming my view that the quantum field approach to quantum gravity can be characterised as a Whiteheadian approach:

"In general relativity […] there is no background geometry. The space-time metric itself is the fundamental dynamical variable. On the one hand, it is analogous to the Minkowski metric in Maxwell's theory […]. On the other hand, it is the analog of the Newtonian gravitational potential […]. This dual role of the metric is in effect a precise statement of the equivalence principle that is at the heart of general relativity. It is this feature that is largely responsible for the powerful conceptual economy of general relativity, its elegance and its aesthetic beauty. However, this feature also brings with it a host of problems. […] It is because there is no background geometry, for example, that it is so difficult to analyze singularities of the theory and to define the energy and momentum carried by gravitational waves. Since there is no a priori space-time, to introduce notions as basic as causality, time, and evolution, one must first solve the dynamical equations and *construct* a space-time. […]

"Field-theoretic techniques are to be applied even if it means sacrificing the geometric beauty of general relativity. Indeed, […] even the dynamics given by Einstein's equations are not to be regarded as sacrosanct. The first step in this program is to split the space-time metric g_{ik} in two parts, $g_{ik} = \eta_{ik} + \gamma h_{ik}$, where η_{ik} is to be a background, kinematical metric, often chosen to be flat, γ is Newton's constant, and h_{ik}, the deviation of the physical metric from the chosen background, the dynamical field. The two roles of the metric tensor are now split. With this splitting most of the conceptual problems discussed above seem to melt away." (Ashtekar 1991:3-4)

After highlighting how the quantum field approach of the first stage of the history of quantum gravity succeeded in treating the gravitational field as just another particle field against the Minkowski background (the graviton field), and how this entails that "one could apply to it all the machinery of perturbation theory that had been so successful in particle physics," Ashtekar comes to the second stage of development, which takes its point of departure in the fact that "detailed calculations revealed that quantum general relativity is perturbatively non-renormalizable at two loops." (Ashtekar 1991:5) In this stage, however, "by the success of perturbative methods in electro-weak interactions, the community was reluctant to give them up in the gravitational case," and "perturbative methods did revive once again in the eighties […] due to another renaissance; that of string theory." (Ashtekar 1991:6-7)

String theory, even though it replaces the familiar particles of quantum field theory by one-dimensional strings, incorporates important aspects of the massless spin-2 particle approach to quantum gravity. Its intention is to unify all fundamental forces in terms of strings in different oscillation-modes, one of which is the massless spin-2 state. However, from a Whiteheadian point of view, string theory is shifting away from the searched for theory because of its mixture of physics and geometry, and because it deals with numerous hidden spatial dimensions which are beyond sense-perception. Moreover, from the physicists point of view, string theory ultimately did not prevent the occurrence of problematic divergences. E.g., Ashtekar writes: "There is general consensus among experts that [string] theory is *finite* – not just renormalizable – order by order in perturbation theory. Unfortunately, however, when summed, the series diverges and does so uncontrollably." (Ashtekar 1991:7) Also, and relevant not only with respect to string theory, but with respect to all perturbative, background dependent theories, "it is far from clear how to incorporate non-perturbative effects, such as those connected with black holes." (Gibbons, personal communication)

Consequently, it will come as no surprise that the third stage in the history of quantum gravity identified by Ashtekar is the search for an alternative to the background dependent and perturbative approaches which did not manage to overcome the appearance of problematic divergences, nor to deal satisfactorily with non-perturbative effects. Loop quantum gravity and M-theory will not be discussed in this paper, but can be taken as characteristic of this third stage in the history of quantum gravity. The reason for not dealing with recent background independent, non-perturbative approaches in this paper is twofold. One: their background independency removes them even further than the original string theory from the Whiteheadian ideal. Two: even though bimetric, background dependent, perturbative theories are forcefully rejected by Einsteinian physicists (see, e.g., Stachel 1991:32-40), no consensus on the superiority of their monometric, background independent, non-perturbative alternatives has been reached within the physics community either. The search for a uniform background approach is not dead. (See, e.g., Pitts & Schieve 2007:700-717)

§7 Summary and conclusion

In his philosophy of nature, Whitehead was inspired by the logic of polyadic relations and by the relativity of space and time. He aimed at replacing the classical model of nature – material particles with physical properties are moving in a space-time container – with a relational model. His general theory of relativity is a theory of relatedness in which spatio-temporal networks of events are characterized by patterns of physical adjectives. In his theory of electromagnetism and gravitation, space-time expresses the uniform relatedness of events, and the physical field expresses the causal relatedness of historical routes of events characterized by definite masses and charges. Space-time finds its mathematical form in the Galilean tensor, the physical field in the impetus tensor. The elements of the first are constant and constitute the Minkowski metric; the elements of the latter are calculated by means of retarded potentials and constitute the equations of motion.

Whitehead's theory of gravitation was most likely inspired by the research of Cunningham and Silberstein, who were also aiming at a relativistic theory of gravitation in which the gravitational field is defined against the background of a uniform space-time, and in terms of retarded potentials. In particular cases – connected with the names of Schwarzschild and Kerr – Whitehead's retarded potentials law to calculate the gravitational impetus gives rise to the exact same equations of motion as does Einstein's law of gravitation, hence entailing empirical equivalence. In other cases, however, as shown by Gibbons and Will, Einstein's theory of gravitation is empirically superior to Whitehead's. Consequently, we are forced to give up Whitehead's particular theory of gravitation.

And yet, the question arises, whether it is not possible to re-interpret Einstein's theory in terms of Whitehead's general theory of relatedness. Whittaker and Herstein have pointed at Rosen's bimetric reinterpretation of Einstein's theory, Palter at Gupta's pioneering spin-2 field reinterpretation of it, and this paper at Feynman's pedagogical spin-2 field reinterpretation. None of these approaches has completely satisfied the physics community. In fact, even today no generally accepted theory of quantum gravity exists. Moreover, even though the Rosen, Gupta, and Feynman, approaches can be labelled as Whiteheadian, I have the impression that they leave out an extremely important aspect of Whitehead's theory: his notion of historical routes, which incorporates – to a certain extent – the Whiteheadian novelty of looking at patterns characterizing networks of events. A truly Whiteheadian approach should not only separate physics and geometry, and it should not only search for a unified treatment of all known fields. It should also make room for a polyadic patterns-events relatedness.

So, I would like to conclude my paper by going beyond its historical overview, and by launching the claim that the philosophical framework of Whitehead might well be the most appropriate framework to guide the imagination of physicists towards a theory of quantum gravity. The latter, of course, is not a justifiable claim. Only dreams can guide us towards the future. According to Isabelle Stengers in *Penser avec Whitehead*, the general theory of relatedness proposed by Whitehead "unfolds a universe of possibilities in the bosom of which the situation described by Einstein […] manifests itself as a very special case." And she adds: "It has been judged by physicists as a useless complication, but now and again I catch myself dreaming that, in the light of the difficulties encountered by contemporary physics when faced with the unification of the gravitational force with the other interactions, this complication might well open interesting perspectives." (Stengers 2002:193)

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