

# The Universe of Willem de Sitter

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## 1 Introduction

The Dutch astronomer Willem de Sitter (1872-1934) gave important contributions to the rise of relativistic Cosmology. The debate from 1916 to 1918 between de Sitter and Albert Einstein (1879-1955) is a fundamental chapter in the history of the scientific view of the universe. In fact, during such a debate both Einstein and de Sitter formulated their own mathematical expressions for the metric of the universe as a whole.

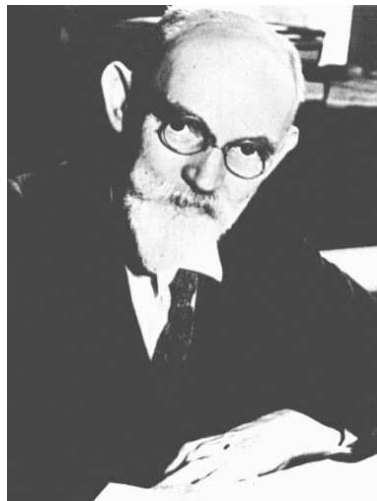


Figure 1: Willem de Sitter (1872-1934) [Photo: Yerkes Observatory].

## 2 Einstein static model of the universe

In the fundamental paper *Kosmologische Betrachtungen zur allgemeinen Relativitätstheorie (Cosmological Considerations in the General Theory of Relativity)*<sup>1</sup>, first published in 1917, Einstein proposed a new interpretation of the universe as a whole. This paper was the result of Einstein's attempts to achieve the "relativity of inertia"<sup>2</sup>, i.e. to obtain the entire material origin of inertia in the framework of General Relativity. In fact, accounting for Mach's ideas, Einstein pointed out that inertia was simply an interaction between masses: "in a consistent theory of Relativity - Einstein stated - there can be no inertia *relatively to 'space,'* but only an inertia of masses *relatively to one another*"<sup>3</sup>.

In the fall of 1916 Einstein debated with de Sitter on the problem of suitable boundary conditions at infinity. According to the Principle of Relativity, Einstein tried to obtain values for the  $g_{\mu\nu}$ 's at infinity that were invariant for all transformations. He avoided this difficulty by replacing such boundary conditions with the condition of closure, introducing a "finite and yet unbounded universe"<sup>4</sup>. Einstein proposed a spherical model of the universe, in which the matter was uniformly and homogeneously distributed.

This static solution had line element:

$$ds^2 = dx_4^2 - g_{\alpha\beta} dx_\alpha dx_\beta; \quad (1)$$

$$g_{\alpha\beta} = - \left[ \delta_{\alpha\beta} + \frac{x_\alpha x_\beta}{R^2 - (x_1^2 + x_2^2 + x_3^2)} \right]. \quad (2)$$

$R$  was the radius of curvature of the three-space  $(x_1, x_2, x_3)$ , that was everywhere orthogonal to the time dimension  $x_4$ .

This model fully achieved the relativity of inertia. There was not any independent property of space which claimed to the origin of inertia, so the latter was entirely produced by masses in the universe. The condition of spatial closure ensured that both the gravitational potential and the hypothetical average density of ponderable matter remained constant *in space*.

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<sup>1</sup>*Einstein 1917b*.

<sup>2</sup>*Einstein 1913*, Engl. tr. p. 197

<sup>3</sup>*Einstein 1917b*, Engl. tr. p. 424.

<sup>4</sup>*Einstein 1917a*, Engl. tr. p. 364.

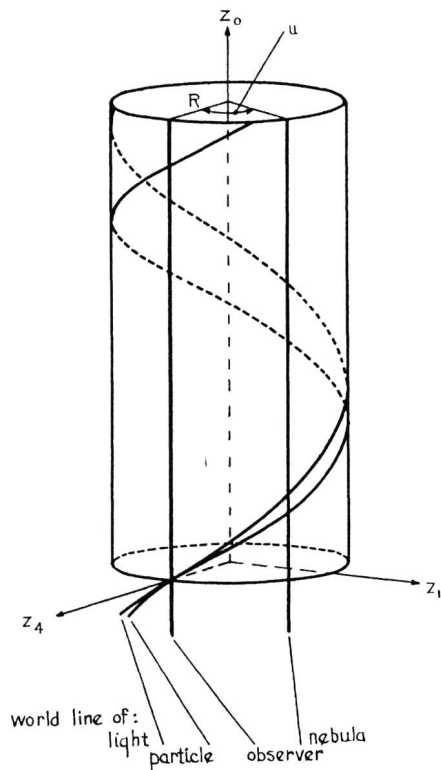


Figure 2: Einstein's "cylindrical" universe. One spatial dimension is disregarded. The vertical axis is along the direction of time [from *Robertson 1933*, p. 70].

Einstein modified his field equations accounting for the supposed static nature of the universe, i.e. to preserve the gravitational potential and the density of matter constant *in time*. He inserted the so-called *cosmological term*, namely the fundamental tensor  $g_{\mu\nu}$  multiplied by  $-\lambda$ , a universal but unknown constant:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = -\kappa T_{\mu\nu}. \quad (3)$$

In this way field equations expressed the observational evidence of the static equilibrium of the universe. In fact, at the beginning of XX Century, no large-scale systematic velocity fields were pointed out in stellar motions by observed data.

The new constant  $\lambda$ , the radius of the universe  $R$ , and the mean density of

“world matter”  $\rho$  were strictly connected:

$$\lambda = \frac{\kappa\rho c^2}{2} = \frac{1}{R^2}. \quad (4)$$

In Einstein model the metric of the universe could be given as a solution of relativistic field equations with the cosmological term. Both the general covariance and the laws of conservation of momentum and energy were still satisfied. According to Einstein, “the burning question whether the relativity concept can be followed through the finish or whether it leads to contradictions”<sup>5</sup> was thus solved. Through this extension of field equations Einstein was “no longer plagued with the problem, while previously it gave no peace”<sup>6</sup>.

### 3 De Sitter “empty” model of the universe

Right after Einstein model appeared, de Sitter proposed his own solution of field equations. The Dutch astronomer admired Einstein conception of the universe “as a contradiction-free chain of reasoning”<sup>7</sup>, and gave a different solution also maintaining the  $\lambda$ -term. However de Sitter preferred the original relativistic theory of gravitation, “without the undeterminable  $\lambda$ , which is just philosophically and not physically desirable”<sup>8</sup>.

#### 3.1 The “mathematical postulate of relativity of inertia”

De Sitter approached the cosmological problem in a different way. It was mainly Paul Ehrenfest (1830-1933) who suggested him a mathematical conception of inertia<sup>9</sup>, which led de Sitter to propose a finite and “empty” universe.

De Sitter was “sceptical”<sup>10</sup> about Einstein cylindrical model, and about the assumption that the world was mechanically quasi-stationary. According

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<sup>5</sup>CPAE 1998, doc. 311.

<sup>6</sup>*Ivi.*

<sup>7</sup>*Ibidem*, doc. 312.

<sup>8</sup>*Ibidem*, doc. 313.

<sup>9</sup>*Ibidem*, doc. 321.

<sup>10</sup>*Ibidem*, doc. 327.

to de Sitter, “all extrapolation is uncertain. (...) We only have a snapshot of the world, and we cannot and must *not* conclude (...) that everything will always remain as at that instant when the picture was taken”<sup>11</sup>. Otherwise “the extrapolation leads us - de Sitter wrote to Einstein - to assume that we have solved a puzzle, when we have just clothed it in other words”<sup>12</sup>. Moreover, the hypothetical average density of “world matter”  $\rho$  was objectionable, because the distribution of stars in the observable portion of the universe was extremely not homogeneous.

De Sitter proposed a distinction between the “world matter” and the “ordinary matter”. The former was hypothetically distributed through space with density  $\rho_0$ . The latter corresponded to observable objects as planets and stars, i.e. to locally condensed world matter with density  $\rho_1$ . By this assumption, de Sitter pointed out that “inertia is produced by the whole of world matter, and gravitation by its local deviations from homogeneity”<sup>13</sup>. Neglecting all pressures and internal forces, and supposing all matter to be at rest, the energy-momentum tensor became:

$$T_{44} = (\rho_0 + \rho_1) c^2 g_{44}. \quad (5)$$

De Sitter made the hypothesis to neglect gravitation on large-scale, and to take  $\rho_0$  constant.

According to de Sitter, the three-dimensional finite world proposed by Einstein satisfied the “material relativity requirement”<sup>14</sup>, or equivalently the “material postulate of relativity of inertia”<sup>15</sup>. This requirement denied the possibility of the existence of a world without matter: “if all matter - de Sitter stated - is supposed not to exist, with the exception of one material point which is used as a test-body, has then this test-body inertia or not? The school of Mach requires the answer *No*. (...) This world matter, however, serves no other purpose than to enable us to suppose it not to exist”<sup>16</sup>.

The Dutch astronomer pointed out that the relativistic field equations

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<sup>11</sup> *Ibidem*, doc. 321.

<sup>12</sup> *Ivi*.

<sup>13</sup> *De Sitter 1917b*, p. 5.

<sup>14</sup> *CPAE 1998*, doc. 321.

<sup>15</sup> *De Sitter 1917b*, p. 5.

<sup>16</sup> *De Sitter 1917a*, p. 1222.

were “the fundamental ones”<sup>17</sup>: the postulate that at infinity all  $g_{\mu\nu}$ ’s were invariant for all transformations was more important than the “Machian” postulate of inertia introduced by Einstein. In fact in Einstein model, for the hypothetical value  $R = \infty$ , the whole of  $g_{\mu\nu}$ ’s degenerated to

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

This set of values was invariant for all transformations for which, at infinity,  $t' = t$ . In other words, in Einstein cylindrical world it was possible to find systems of reference in which the  $g_{\mu\nu}$ ’s only depended on the space-variables, and not on the “time”. However the “time” of such a systems had “a separate position”<sup>18</sup>, because it was “the same always and everywhere”<sup>19</sup>. For such a reason, according to de Sitter, the time coordinate in Einstein model was nothing else than an absolute time, and there the world matter took “the place of the absolute space in Newton’s theory, or of the inertial system”<sup>20</sup>. De Sitter proposed that the potentials should have degenerate at infinity to the values:

$$\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

According to him, “if at infinity all  $g_{\mu\nu}$ ’s were *zero*, then we could truly say that the *whole* of inertia, as well as gravitation, is thus produced. This is the reasoning which has led to the postulate that at infinity all  $g_{\mu\nu}$ ’s shall be zero”<sup>21</sup>. De Sitter called this requirement the “mathematical relativity condition”<sup>22</sup>, or the “mathematical postulate of relativity of inertia”<sup>23</sup>. In

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<sup>17</sup> *De Sitter 1916b*, p. 529.

<sup>18</sup> *De Sitter 1917a*, p. 1223.

<sup>19</sup> *De Sitter 1917b*, p. 11.

<sup>20</sup> *Ibidem*, p. 9.

<sup>21</sup> *Ibidem*, p. 4.

<sup>22</sup> *CPAE 1998*, doc. 321.

<sup>23</sup> *De Sitter 1917b*, p. 5.

fact, such a condition corresponded to the possibility that “the world as a whole can perform random motions without us (within the world) being able to observe it”<sup>24</sup>: “the postulate of the invariance of the  $g_{\mu\nu}$ ’s at infinity - de Sitter stated - has no real physical meaning. It is purely mathematical”<sup>25</sup>.

### 3.2 A universe without “world matter”

In a letter to Einstein<sup>26</sup> de Sitter proposed his own solution of the metric of the universe as a whole, actually the second relativistic model in modern Cosmology.

The Dutch astronomer considered field equations with the  $\lambda$ -term and without matter, i.e. by assuming  $\rho_0 = 0$ :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R - \lambda g_{\mu\nu} = 0. \quad (6)$$

These equations could be satisfied by the  $g_{\mu\nu}$ ’s given by the metric:

$$ds^2 = \frac{-dx^2 - dy^2 - dz^2 + c^2 dt^2}{[1 - \frac{\lambda}{12}(c^2 t^2 - x^2 - y^2 - z^2)]^2}. \quad (7)$$

The coordinates  $(x, y, z, t)$  could have infinite values, on condition that  $g_{\mu\nu}$ ’s were null at infinity. Such a condition was equivalent to the finiteness of the world in natural (proper) measure. In fact the length of any semi-axis in natural measure was:

$$L_\alpha = \int_0^\infty \sqrt{-g_{\alpha\alpha}} dx_\alpha. \quad (8)$$

A finite world (i.e. a finite value of  $L_\alpha$ ) necessary implied  $g_{\alpha\alpha} = 0$  for  $x_\alpha \rightarrow \infty$ , and vice versa<sup>27</sup>.

De Sitter pointed out that in his model no world matter was necessary, and the insertion of the  $\lambda$ -term satisfied the mathematical postulate of relativity of inertia. In this system there was not any universal time, nor any difference between the “time” and the other coordinates: none of these coordinates had any physical meaning<sup>28</sup>. The cosmological constant determined

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<sup>24</sup>CPAE 1998, doc. 321.

<sup>25</sup>De Sitter 1917a, p. 1223.

<sup>26</sup>CPAE 1998, doc. 313.

<sup>27</sup>De Sitter 1917b, p. 5.

<sup>28</sup>Ibidem, p. 11.

the value of the curvature radius  $R$ :

$$\lambda = \frac{3}{R^2}. \quad (9)$$

By using an imaginary “time”-coordinate  $\xi_4 = ict$ , the geometry of de Sitter world was that of a 4-dimensional hyper-sphere which could be described in a 5-dimensional Euclidean space:

$$R^2 = \xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \xi_5^2. \quad (10)$$

In hyper-spherical coordinates the metric of such a *four-dimensional world* resulted:

$$ds^2 = -R^2\{d\omega^2 + \sin^2\omega[d\zeta^2 + \sin^2\zeta(d\psi^2 + \sin^2\psi d\theta^2)]\}, \quad (11)$$

where  $0 \leq \theta \leq 2\pi$ ;  $0 \leq \psi, \zeta, \omega \leq \pi$ . Equivalently, by replacing the imaginary “time”-coordinate  $\xi_4$  with a real time-coordinate ( $\xi_4 \rightarrow i\xi_4$ ), the geometry of de Sitter world corresponded to a 4-dimensional hyperboloid in a 4+1-dimensional Minkowski space-time:

$$R^2 = \xi_1^2 + \xi_2^2 + \xi_3^2 - \xi_4^2 + \xi_5^2. \quad (12)$$

By pseudo-spherical coordinates (with  $i\omega' = \omega$ ), the metric of space-time resulted:

$$ds^2 = R^2\{d\omega'^2 - \sinh^2\omega'[d\zeta^2 + \sin^2\zeta(d\psi^2 + \sin^2\psi d\theta^2)]\}, \quad (13)$$

where  $0 \leq \theta \leq 2\pi$ ;  $0 \leq \psi, \zeta \leq \pi$ ;  $-\infty < \omega' < +\infty$ .

The potentials in the hyper-spherical coordinate system were:

$$g_{\mu\nu} = - \left[ \delta_{\mu\nu} + \frac{x_\mu x_\nu}{R^2 - (x_1^2 + x_2^2 + x_3^2 + x_4^2)} \right]. \quad (14)$$

Thus the metric proposed by de Sitter,

$$ds^2 = \frac{-dx^2 - dy^2 - dz^2 + c^2 dt^2}{[1 - \frac{\lambda}{12}(c^2 t^2 - x^2 - y^2 - z^2)]^2}, \quad (15)$$

could be obtained by the stereographic projection of the 4-dimensional hyper-sphere to the Euclidean space, or equivalently by the projection of the hyperboloid to a 3+1-dimensional Minkowski space-time<sup>29</sup>.

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<sup>29</sup>*Ibidem*, p. 10.

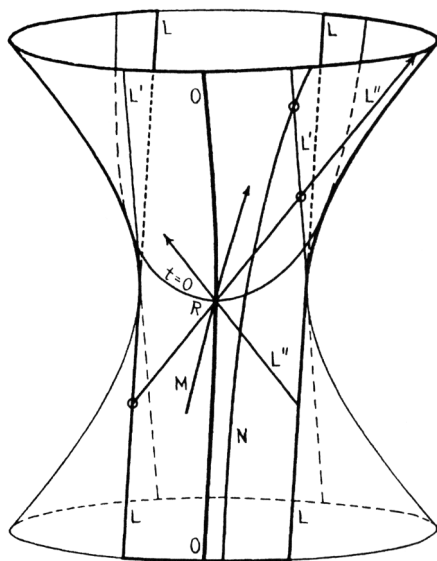


Figure 3: The de Sitter universe. Two spatial dimensions are suppressed [from *Robertson 1933*, p. 71].

“If a single test particle - de Sitter wrote to Einstein - existed in the world, that is, there were *no* sun and stars, etc., it would have inertia”<sup>30</sup>. Essentially, in the universe proposed by de Sitter a suitable metric was obtained without any “physical” masses<sup>31</sup>. Such forms of matter as stars and nebulae were to be regarded as “test particles” in a fixed background metric, which curvature was determined by the cosmological constant<sup>32</sup>.

### 3.2.1 Einstein criticism

Einstein acknowledged de Sitter’s solution to be “very interesting”<sup>33</sup>, but “must have been disappointed”<sup>34</sup>, and tried to discard this anti-Machian solution: “I cannot grant - Einstein wrote to de Sitter - your solution any physical possibility”<sup>35</sup>. In fact, the cosmological term took a fundamental

<sup>30</sup> *CPAE 1998*, doc. 313.

<sup>31</sup> *Ivi.*

<sup>32</sup> *Bernstein-Feinberg 1986*, p.10.

<sup>33</sup> *CPAE 1998*, doc. 317.

<sup>34</sup> *Pais 1982*, p. 287.

<sup>35</sup> *CPAE 1998*, doc. 366.

role in de Sitter model in order to involve a sort of spatial (and not material) origin of inertia. “The  $g_{\mu\nu}$  field - Einstein replied to de Sitter - should be fully determined by matter, and not be able to exist without the latter”<sup>36</sup>.

At first Einstein objected that the hyperboloid surface

$$1 - \frac{\lambda}{12}(c^2t^2 - x^2 - y^2 - z^2) = 0 \quad (16)$$

was a singularity. On this surface there was a discontinuity, because the  $g_{44}$  term “jumped”<sup>37</sup> from  $+\infty$  to  $-\infty$ , and  $g_{\alpha\alpha}$ ’s from  $-\infty$  to  $+\infty$ . Such a surface lied in the physically finite, but it was not possible to assume infinite values for the potentials, because of the supposed static nature of the universe and the small velocities measured on stars<sup>38</sup>. Moreover, the four-dimensional continuum proposed by de Sitter did not have the property that all its points were equivalent<sup>39</sup>. In fact it had a preferred point, i.e. the center of the conic section

$$1 + (c^2t^2 - x^2 - y^2 - z^2) = 0. \quad (17)$$

De Sitter replied that the hyper-surface involved a finite natural spatial distance and an infinite natural temporal distance. Thus the discontinuity was only apparent, and this problem was “not interesting”<sup>40</sup>. Also the supposed preferred point was later shown to be a geometrical consequence of that choice of coordinates, and not a true physical aspect<sup>41</sup>. “My four-dimensional world - de Sitter remarked to Einstein - also has the  $\lambda$ -term, but no *world matter*”<sup>42</sup>.

### 3.2.2 Elliptical geometry

In order better to compare his own model with Einstein solution, de Sitter proposed another expression of the metric<sup>43</sup>. By using spherical polar

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<sup>36</sup>*Ibidem*, doc. 317.

<sup>37</sup>*Ivi*.

<sup>38</sup>*Ibidem*, doc. 351.

<sup>39</sup>*Ivi*.

<sup>40</sup>*Ibidem*, doc. 327.

<sup>41</sup>*Ibidem*, doc. 363.

<sup>42</sup>*Ivi*.

<sup>43</sup>*Ibidem*, doc. 355.

coordinates, he represented the hyperboloid universe (system  $B$ ) as the Einstein universe (system  $A$ ), i.e. as 3-dimensional hyper-spheres embedded in a 4-dimensional Euclidean space:

$$ds_A^2 = -dr^2 - R^2 \sin^2 \frac{r}{R} (d\psi^2 + \sin^2 \psi d\theta^2) + c^2 dt^2, \quad (18)$$

$$ds_B^2 = -dr^2 - R^2 \sin^2 \frac{r}{R} (d\psi^2 + \sin^2 \psi d\theta^2) + \cos^2 \frac{r}{R} c^2 dt^2. \quad (19)$$

De Sitter pointed out that, between the possible forms of space with constant curvature, the elliptical space was more preferable than the spherical one<sup>44</sup>. In the elliptical space, which also was closed respect to its dimensions, any two straight lines could not have more than one point in common.<sup>45</sup> Einstein agreed with de Sitter on the choice of elliptical space<sup>46</sup>, but he noticed that the spherical geometry he used in the *Cosmological Considerations in the General Theory of Relativity* only was an approximation. According to Einstein, it served to show “through an idealization, that a spatially closed (finite) system is possible. (...) The system could actually be quite irregularly curved, also on a large scale, that is, it could relate to the spherical world like a potato’s surface to a sphere’s surface”<sup>47</sup>.

By the new expression of the line element (the so called *static form*) it was clear that all the points in de Sitter world were equivalent<sup>48</sup>. However, as Einstein pointed out, the  $g_{44}$  coefficient of the temporal term in system  $B$  depended on position. In fact, being  $g_{44} = \cos^2(\frac{r}{R})$ , such a potential changed its value from 1 (for  $r = 0$ ) to 0 (for  $r = \frac{\pi}{2}R$ ). According to Einstein, time clocks slowed down approaching  $r = \frac{\pi}{2}R$ : this null value for the potential involved that all masses had the tendency to aggregate at this “equator”<sup>49</sup>. “It seems - Einstein wrote in 1918 - that no choice of coordinates can remove this discontinuity. (...) We have to assume that de Sitter solution has a genuine singularity on the surface  $r = \frac{\pi}{2}R$  in the finite

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<sup>44</sup>*Ivi.* De Sitter noticed that already in 1900 Karl Schwarzschild (1873-1916) studied some astronomical consequences of elliptical and hyperbolic spaces.

<sup>45</sup>*De Sitter 1917b*, p. 8.

<sup>46</sup>*CPAE 1998*, doc. 359.

<sup>47</sup>*Ibidem*, doc. 356.

<sup>48</sup>*Ibidem*, doc. 363.

<sup>49</sup>*Ivi.*

domain. (...) The de Sitter system does not look at all like a world free of matter, but rather like a world whose matter is concentrated entirely on the surface  $r = \frac{\pi}{2}R$ <sup>50</sup>. According to Einstein, a *free of matter* solution of field equations was inconceivable. Through his critical comment to de Sitter solution, Einstein noticed that the cosmological constant did not involve any sort of spatial origin of inertia<sup>51</sup>.

De Sitter acknowledged Einstein remark to be correct, but gave a different interpretation. According to the Dutch astronomer, such a remark involved a *philosophical*, and not a *physical* requirement<sup>52</sup>. In fact, the “equator” at  $r = \frac{\pi}{2}R$  was at a finite distance in space, but was physically inaccessible<sup>53</sup>. The velocity of a material particle became zero for  $r = \frac{\pi}{2}R$ . Thus a material particle which was on the polar line on the origin could have no velocity, nor energy. “All these results - de Sitter stated - sound very strange and paradoxical. They are, of course, all due to the fact that  $g_{44}$  becomes zero for  $r = \frac{\pi}{2}R$ . We can say that on the polar line the four-dimensional time-space is reduced to the three-dimensional space: *there is no time*, and consequently no motion”<sup>54</sup>. The time needed by a ray of light, or by a material particle, to travel by any point to the equator was infinite. Thus the singularity at  $r = \frac{\pi}{2}R$  could never affect any physical experiment<sup>55</sup>.

The issue was solved by Felix Klein (1849-1925). In some correspondence with Einstein, the authoritative mathematician showed that the singularity

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<sup>50</sup>*Einstein 1918b*, Engl. tr. p. 37.

<sup>51</sup>It is interesting to note that Einstein at the same time proposed both such a critical comment on de Sitter solution (*Einstein 1918b*) and a new paper on the foundations of General Relativity (*Einstein 1918a*). In the latter, Einstein wanted to underline the importance of three independent aspects on which the theory is based. Together with the Principle of Relativity and the Principle of Equivalence, he took into account a third aspect, i.e. “Mach’s Principle”: “The  $G$ -field is completely determined by the masses of the bodies. Since mass and energy (...) are the same, and since energy is formally described by the symmetric energy tensor ( $T_{\mu\nu}$ ), it follows that the  $G$ -field is caused and determined by the energy tensor of matter” (*Ibidem*, Engl. tr. p. 33.). Thus, what Einstein had called the “relativity of inertia” became one of the fundamental principles of General Relativity.

<sup>52</sup>*CPAE 1998*, doc. 501.

<sup>53</sup>*De Sitter 1918*, p. 1309.

<sup>54</sup>*De Sitter 1917b*, p. 17.

<sup>55</sup>*De Sitter 1918*, p. 1309.

at the equator in de Sitter universe could be eliminated by using the first coordinates of the hyperboloid form. Thus such a singularity could “simply be transformed away”<sup>56</sup>: it only was a geometrical consequence of the choice of coordinates<sup>57</sup>. The matter-free model proposed by de Sitter was free of singularities, and its space-time points were all equivalent. At the end Einstein admitted that de Sitter solution existed. However he still believed that this “anti-Machian” universe was not a physical possibility<sup>58</sup>.

## 4 Conclusion

“At the present time - de Sitter wrote in 1920 - the choice between the systems *A* and *B* is purely a matter of taste. There is no physical criterion as yet available to decide between them”<sup>59</sup>. However de Sitter noticed that these systems differed in their physical consequences. In fact, in de Sitter world a particle at rest would not have remained at rest unless it was at the origin. This mass test would have escaped far away because of the presence of the cosmological constant. Thus the de Sitter system to all appearances was static, and required a positive radial velocity for distant objects. This effect of recession was known as “de Sitter effect”. At that time, and during the Twenties, this effect appeared to be connected in some manner with the first red-shift observations of many nebulae. The interest in de Sitter effect survived until 1930, when truly non-static theoretical models of the universe were proposed to explain the red-shift problem and the astronomical evidences of a cosmic recession<sup>60</sup>.

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<sup>56</sup>CPAE 1998, doc. 566.

<sup>57</sup>For a detailed report on the Einstein-Klein discussion of de Sitter cosmological model see *Janssen 2003*.

<sup>58</sup>CPAE 1998, doc 567.

<sup>59</sup>*De Sitter 1920*, p. 867.

<sup>60</sup>For a comprehensive study about the de Sitter effect see *North 1965*, pp. 92-109.

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