IMPLICIT RESTRICTED VALIDITY OF CONCEPTS IN LORENTZ FORMULAE TO LIMITED SPACE-TIME INTERVALS

Cristian I. Toma

Department of Applied Informatics, Titu Maiorescu University, Bucharest

Abstract. Certain intuitive problems connected with measurements on closed-loop trajectories in special relativity and non-commutative properties of operators in quantum physics imply a more rigorous definition of measurement method and of the interaction phenomena, classified from the wave and from the corpuscular aspect of matter, so as to avoid contradictions. The use of least action priciple implies also some logic definitions for measuring methods based on the waves and for measuring methods based on the corpuscular aspecst of matter, starting from considerations about a possible memory of previous measurements (operators) in case of a sequence of received pulses. Due to this, it results a certain distinction between the set of existing space-time intervals (which can be defined on unlimited space-time intervals) and the set of measured space-time intervals (established using measuring methods based on waves and always defined on limited space-time intervals)..

1. Introduction

As it is known, basic concepts in physics connected with interaction are the wave and corpuscle concepts. In classical physics the corpuscle term describes the existence of certain bodies subjected to external forces or fields, and the wave concept describes the propagation of oscillations and fields (sometimes equations able to generate practical test functions[1] being used) In quantum physics, these terms are closely interconnected, the wave train associated to a certain particle describes the probability of a quantum corpuscle (an electron or a photon) to appear; the results of certain measurements performed upon the quantum particle are described by the proper value of the operators corresponding to the physical quantity to be measured. Certain intuitive problems connected with measurement procedures on closed-loop trajectories in special relativity and non-commutative properties of operators in quantum physics[2] imply a more rigorous definition of measurement method and of the interaction phenomena, classified from the wave and from the corpuscular aspect of matter, so as to avoid contradiction generated by terminological cycles[3]. Logic definition for the class of measuring methods based on the wave aspect of matter and for the class of measuring methods based on the corpuscular aspect of matter upon interaction phenomena, based on considerations about a possible memory of previous measurements (operators) in case of a sequence of received pulses were presented in previous papers[4], trying to obtain expressive pattern classes[5].

2. The Necessity for Associating a Wave-Function to the Lorentz Transformation

The Lorentz transformation is usually represented as a matrix L which acts upon a cuadridimensional column vector r having the components $r_1=x$, $r_2=y$, $r_3=z$, $r_4=ict$, resulting another cuadridimensional vector r′ having the components $r'_1=x'$, $r'_2=y'$, $r'_3=z'$, $r'_4=ict'$, where x, y, z, t are the space-time coordinates corresponding to a certain event in an inertial reference system S, and x′, y′ , z′ , t′ are the space-time coordinates corresponding to the same event measured in an inertial reference system S′ which moves with velocity v (a vector) as against the system S. This means

$$r'=L(v)\, r$$

All time moments are considered after a synchronization moment (when the clock indications in the reference systems are set to zero). The velocity v defines the matrix L, and the result is considered not to depend on the measuring method used.

But let us consider that the velocity v has two components v_x and v_y oriented along the Ox axis (for v_x) and along the Oy axis (for v_y) and let us consider also that the event taking place in the reference system S is first observed in a reference system S_1 which moves with velocity v_x as against the system S: a set of space-time coordinates (x_1, y_1, z_1, t_1) will be established for the event. Then the event having the space-time coordinates (x_1, y_1, z_1, t_1) in system S_1 is observed in the reference system S′ which moves with velocity v_y (the projection of v along the Oy axis) against the reference system S (this relative speed being measured in system S). That corresponds to a relative speed

$$v_y(c)=v_y/(1-v_x^2/c^2)^{-1/2}$$

between the systems S and S′ (due to the kinematics law of addition of speeds in special relativity theory). Thus will result the cuadridimensional vector r′ (having the components x′, y′, z′, ict′) , measured in system S′, under the form

$$r'=L(v_y(c))\, L(v_x)\, r$$

But we can also consider that the event having the space-time coordinates x, y, z, t in system S is first observed in a reference system S_2 which moves with velocity v_y (the projection of velocity v along the Oy axis) as against system S; a set of space-time coordinates will be established for the event. Then this event having the space-time coordinates x_2, y_2, z_2, t_2 in system S_2 is observed in the reference system S' which moves with velocity v_x (the projection of velocity v along the Ox axis) as against the reference system S_2, the velocity v_x being measured in the reference system S. That corresponds to a relative speed

$$v_x(c)=v_x/(1-v_y^2/c^2)^{-1/2}$$

between the systems S' and S_2 (due to the same kinematics law of addition of speeds in special relativity). Thus will result the space-time coordinates x', y', z', t' measured in system S' under the form

$$r'=L(v_x(c))\ L(v_y)\ r$$

Using the explicite form of Lorentz transformation for the case when the relative speed has the direction of one of the axes of coordinates, it can be easy shown that

$$L(v_y(c))\ L(v_x)\ r \neq L(v_x(c))\ L(v_y)\ r$$

This shows that the coordinates measured for the event in S' reference system depends on the succession of transformations.

This aspect is similar to the non-commutative properties of operators in quantum theory. It implies that in the case of special relativity we must presents to the students the ncessity to define a vector of state (a wave-function).upon which the Lorentz transformation acts.. Thus the Lorentz transformation can be considered by the MS students as a physical transformation which modifies a certain wave-function inside a reference system. Taking into account the fact that usually we receive informations under the form of electromagnetic (or light) wave-trains (the emission of these wave-trains corresponding to the event)and taking also into account the fact that the time-dilation phenomenon (a consequence of Lorentz transformation) was first time observed for light wave-trains (the transverse Doppler effect) it results that in the most general case this wave-function must be associated to the wave-function of the received light wave-train.

As a consequence of the previous statement, it results that a Lorentz transformation L must be always put in correspondence with a pair (S, φ), S representing a certain material reference system which acts upon a wave-train having the state-vector φ. So the Lorentz transformation must be written under the form L_S (φ); in the most general case L is the Lorentz matrix and φ is a vector or a higher -order tensor which describes the field. For an electromagnetic wave, the field can be described using the cuadridimensional vector A. The action of the matrix L_S consists in a general transformation $\mathbf{L_S}$

$$\varphi\ (x, y, z, t) \rightarrow \varphi'\ (x', y', z', t') = \mathbf{L_S}\ \varphi\ (x, y, z, t)$$

where the values of φ are modified according to the transformation rules of vectors and tensors (for example, A'=LA for an electromagnetic wave described by the cuadrivector A) and in the change of the space-time coordinates (x, y, z, t) into (x', y', z', t') according to the formula

$$[x']=L_S[x]$$

[x] representing the cuadridimensional vector of coordinates. We have to point the fact that in all these formulae φ (x, y, z, t) represents the value φ would have possessed in the absence of the interaction with the observer's material medium; the space-time origin must be considered in the point of space and at the moment of time where the wave first time interacts with the observer's material medium (in a similar way with the aspects in quantum mechanics, where all transformations are acting after the interaction with the measuring system).

This interpretation can solve the contradictions appearing in case of movements on closed-loop trajectories (the twins paradox) in a very simple manner. The Lorentz transformation being a transformation which acts upon a certain wave-train (a light wave-train, in the most general case), it has no consequences upon the age of two observers moving on closed-loop trajectories. So no contradiction can appear when the two observers are meeting again.

2. Possibilities of Using the Principle of Least Action in Connection with the "Wave-train" Interpretation

For rejecting the last students' hesitations for accepting this intuitive interpretation, aspects connected with the use of the cuadri-dimensional interval and with the principle of leat action have been also presented The students are already accustomed with basic notions in analytical mechanics; thus we can use the principle of least action for all problems connected with optical wave-trains measurements performed by different observers.

We begin by writing the propagation equation for an electromagnetic wave inside an observer's material medium under the form $dx^2 + dy^2 + dz^2 = c^2t^2$ (c representing the light speed). It results that $c^2t^2 - x^2 - y^2 - z^2 = 0$ for all points inside the material medium where the wave has arrived. But

$$c^2t^2 - x^2 - y^2 - z^2 = ds^2$$

where ds is the cuadridimensional space-time interval. The propagation equation of the optical wave can be written as ds = 0, and so it results that the trajectory of the wave inside the material medium between two points a and b is determined by the equation

$$\int_a^b ds = \Delta s = 0$$

By the other hand, for mechanical phenomena the quantity determining the trajectory of a material body inside a reference system is the action S. Under a relativistic form, it can be written as $S = -mc\int_a^b ds$ m representing the mass of the body, and a, b - the space-time coordinates for two points situated along the "universe line" on which the body moves. The principle of least action can be written as $\delta S = -mc\delta\int_a^b ds = 0$ While $\delta S = \Sigma_i mcu_i\delta x_i$ (where $u_i = v_i/(1-v^2/c^2)^{1/2}$ for i = 1, 2, 3 and $u_4 = ic/(1-v^2/c^2)^{1/2}$) it results finally that $\Sigma_i p_i^2 = -m^2c^2$, p_i being the cuadrivector $\partial S/\partial x_i$ (the momentum). For a free particle, $p_i = mu_i$.

It can be noticed that the infinite small cuadridimensional interval ds is used both for describing the propagation of an electromagnetic wave and the movement of a body inside a reference system. While is it related to the action S, this result is easy to be understood (the principle of least action being a basic principle in nature).

The next step consists in pointing the fact that the previous integral $\Delta s = 0$ (determining the trajectory of the optical wave-train inside the material medium) is based on the supposition that both points a, b belong to the material medium (otherway the velocity of the wave may differ, depending on the dielectric and magnetic constants of the material). So the equation can be directly used in measurement procedures (for establishing trajectory or other properties of the wave only for the time interval when the optical wave-train exists in that material medium.

If an observer has to analyze a wave-train emitted in another material reference system, he must use the invariance property of the cuadriinterval: ds = ds', where ds represents the cuadriinterval between two close events in a certain inertial reference system and ds' represents the cuadriinterval between the same two events measured in another reference system. While ds = ds (dx, dy, dz, dt) is determined inside the observer's reference system and ds' = ds' (dx', dy', dz', dt') corresponds to the reference system where the wave has been emitted, it results that the cuadridimensional interval ds moves into the cuadridimensional interval ds' by a function

$$ds\ (dx, dy, dz, dt) \Rightarrow L \Rightarrow ds'\ (dx', dy', dz', dt')$$

where the arguments of ds are transformed by the Lorentz relations

$$dx' = (dx+vdt)/(1-v^2/c^2)^{1/2},\ dy'=dy,\ dz'=dz,\ dt'=(dt+vdx/c^2)/(1-v^2/c^2)^{1/2}$$

for v parallel to Ox (all the space and time intervals dx, dy, dz and dt being considered inside the observer's material medium after the emitted optical wave-train arrives), and ds = ds'. The above relation can be considered as presenting a transformation of the received wave-train (with x, y, z, t coordinates) into a "supposed" wave-train corresponding to the case when the wave-train wouldn't have entered inside the observer's material medium. For determining the real trajectory of the wave before interaction the observer must extend the trajectory of the received wave-train (having coordinates x', y', z', t') in the past and outside the observer's material medium, using the relation

$$\int_a^b ds' = \Delta s' = 0$$

4. Aspects connected with measurements on closed-loop trajectories

If the Lorentz formulae are considered to be valid at any moment of time after a certain synchronization moment (the zero moment) irrespective to the measuring method used, then certain intuitive problems appear in case of movements on closed-loop trajectories. For example, we can suppose that at the zero moment of time, in a medium with a gravitational field which can be neglected (the use of the galileean form of the tensor g_{ik} being allowed) two observers are beginning a movement from the same point of space, in opposite directions, on circular trajectories having a very great radius of curvature. After a certain time interval, the observers are meeting again in the same point of space.

For very great radii of curvature, the movements on very small time intervals can be considered as approximative inertial (as in the case of the transverse Doppler effect, where the time dilation phenomenon was noticed in the earth reference system which is approximative inertial on small time intervals). The Lorentz formulae can be applied on a small time interval $\Delta t(1)$ measured by one of the observers inside his reference system S_1, and it results (using the Lorentz formula for time) that this interval corresponds to a time interval $\Delta t'(1) > \Delta t(1)$ in the reference system S_2 of the other observer, which moves with speed $v(1)$ as related to the reference system S_1 on this time interval. So the time dilation phenomenon appears. If each observer considers the end of this time interval ($\Delta t(1)$ or $\Delta t'(1)$) as a new zero moment (using a resynchronization procedure), the end of the second time interval $\Delta t(2)$ (with the new zero moment considered as origin) will correspond to a time moment $\Delta t'(2) > \Delta t(1)$ measured in the other reference system S_2 which moves with speed v(2) as related to system S_1 on the time interval $\Delta t'(2)$ (with the new zero moment considered as origin). As related to the first zero moment (when the circular movement has started) the end of the second time interval appears at the time moment $t\{2\} = \Delta t(1) + \Delta t(2)$ for the observers situated in reference system S_1, and at a time moment $t'(2) = \Delta t'(1) + \Delta t'(2) > \Delta t(1) + \Delta t(2)$ $ for the other observer. Thus a global time dilation for the time interval $\Delta t(1) + \Delta t(2)$ appears. The procedure can continue till the end of the whole circular movement (noted as T in system S_1 and T' in system S_2), and by joining together all these time intervals $\Delta t(i)$ we obtain that T'> T. But the whole procedure can be applied starting from another set of small time intervals $\Delta t'(i)$ considered in the reference system S_2 which corresponds to a new set of time intervals $\Delta t(i)$ considered in the reference system S_1 (established using the same Lorentz relation) and finally it would result that the period of the circular movement T' measured in system S_2 corresponds to a period T greater than T' considered in reference system S_1. Contradiction appears in case of measurements based on mechanical or biological methods for measuring time can't be avoided (as statement: a metalic plate is younger as another plate, so it has a greater mechanical resistance and can destroy the other, but in the same time it is older than the other, so it has a less mechanical resistence and can be destroyed by the other when the observers are meeting again). So the Lorentz transformation should be associated with a transformation of a certain wave train when it interacts with the observer's material medium, and this interpretation can be extended at wave-trains associated to particles in quantum physics[7].

Moreover, this aspect implies an intuitive interpretation for the dependence of the mass of a body inside a reference system. Thus, it was shown that for the case when the Lorentz transformation doesn't generate a pulse (for example when the relative speed between the material body and the wave is equal to c, the speed of light in vacuum), the mass m is equal to ∞, which means that no interaction due to the received pulse exists. This manner the notion on infinite mass is connected with the absence of interaction[8]. So $m = \infty$ for a body inside a reference system S shows that we can't act upon the material body using wave pulses emitted in system S; however, changes in the movement of the body (considered in system S) due to other external forces seem to be allowed. The absence of interaction is connected also with absence of estimation for space coordinates of the wave source[9]. This aspect can be considered as a suddenly emerging phenomenon, while the interaction disappears when the relative speed v between the system which emits the wave and the system which receives it becomes equal to c (see related paper[6]).

5. Conclusions

As a major consequence in definition of concepts for reference systems, it results a certain distinction between the set of existing space-time intervals (which can be defined on unlimited space-time intervals) and the set of measured space-time intervals (established using measuring methods based on electromagnetic or associated waves and always defined on limited space-time intervals corresponding to the wave-train transformed by the material medium of the observer's reference system).

References

{1} Toma, G. : Practical test-functions generated by computer algorithms, Lecture Notes Computer Science vol. 3482 (2005), 576--585

{2} Toma, C.: The advantages of presenting special relativity using modern concepts, Balkan Physics Letters Supplement 5 (1997), 2334--2337

{3} Baader, F. : The instance problem and the most specific concept in the description logic w.r.t. terminological cycles with descriptive semantics, Lecture Notes in Computer Science vol. 2821 (2003) 64--78

{4} Toma, C.: A connection between special relativity and quantum theory based on non-commutative properties and system - wave interaction, Balkan Physics Letters Supplement 5 (1997), 2509--2513

{5} Takeda, M., Inenaga, Sh., Bannai, H.: Discovering most classificatory patterns for very expressive pattern classes, Lecture Notes in Computer Science vol. 2843 (2003) 486--493

{6} Toma, C., Pusca, St., Morarescu, C:: Spatial Aspects of Interaction between High-Energy Pulses and Waves Considered as Suddenly Emerging Phenomena vol.3980 (2006) - to appear

{7} Sterian, P., Toma C.: Methods for presenting key concepts in physics for MS students by Photon-MD program, Bulgarian Journal of Physics vol. 27, 4 (2000), 27--30

{8} Simeonidis, M., Pusca, St., Toma, G., Toma, A., Toma, Th.: Definition of wave-corpuscle interaction suitable for simulating sequences of physical pulses, Lecture Notes in Computer Science vol. 3482, (2005), 569--576

{9} Toma, C.: About some space relations of the electromagnetic interaction,
Revue Roumaine des Sciences Techniques, Serie Electrotechnique et Energetique, {\bfseries 34}, 3 (1989), 431--435