"Big Bang" versus "Steady State"

The γ-factor as Arbiter in a New Contest

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Summary

In the present paper it is shown how it is possible to use the strict "light principle" as a point of departure for deriving three new "steady state" models of the universe which are at variance with the Robertson Walker Metric but fulfil Milne's cosmological principle.

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A. *Introduction.*

The γ-factor, which is defined as the quotient between an element of frame time dt and an element of proper time dT, is the most noticeable consequence of special relativity (SR).

A standard clock passing along a series of slave clocks, distributed spatially over the co-moving ("stationary") frame of an observer and synchronized in the conventional way to the master clock of that observer, will thus appear retarded according to: $\gamma = dt/dT = \sqrt{1-dr^2/dt^2}$.

On the other hand, $dT^2 = dt^2 - dr^2 = dt'^2 - dr'^2$ is usually seen as a direct consequence of the differential Lorentz Transformations (LT') in accordance with the relativity principle (RP) and the principle of a constant light speed, here termed the "light principle" (LP).

How can dT be delayed relative to dt and yet be invariant? In order to solve this problem we must proceed to cosmology. One of the first "big bang" (BB) models and the only one based on the integral Lorentz Transformations (LT) is the uniform expansion model of E.A. Milne, cf. his [1935,1958]. In a paper on Milne's theory of kinematic relativity (KR), his former student A.G. Walker [1937] showed how the model can be restated so as to accomodate a cosmic time, $\widehat{\mathcal{T}}$:

(A.1)
$$t = \mathcal{T} ch\sigma \ , \ r = \mathcal{T} sh\sigma$$
$$d\widehat{\mathcal{T}}^{\,2} = dt^2 - dr^2 = d\mathcal{T}^2 - \mathcal{T}^2 d\sigma^2$$
$$d\sigma \to 0 \Rightarrow d\mathcal{T} \to d\widehat{\mathcal{T}}$$

This result inspired him to construct a method for the development of models incompatible with LT, and thereby to develop his own version of the famous Robertson-Walker metric (RWM):

(A.2)
$$d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - \mathcal{S}^2(\mathcal{T})\, d\sigma^2 = \mathit{invar}.$$

Here $\mathcal{T}$ is a universal parameter, $\mathcal{S}(\mathcal{T})$ is a universal scale factor, and σ is a co-moving coordinate characterizing one fundamental observer relative to another.

It seems probable that the Milne model is unique in the manner that it is the only model to combine LP in the strict sense with RWM. Thus, when Bondi & Gold presented their "steady state" (SS) model they explicitly based it on RWM, noticing that the model is incompatible with LT.

The scale factor of an SS universe being $\mathcal{S}_1(\mathcal{T}) \equiv e^{\mathcal{T}}$, it is easy to demonstrate that the model of Bondi & Gold with standard definitions of t & r is incompatible with the strict LP:

(A.3)
$$d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - e^{2\mathcal{T}} d\sigma^2 = (dt^2 - dr^2)(1 - th^2 r)$$

This is shown in the appendix where also two other models with scaling functions $\mathcal{S}_2(\mathcal{T}) \equiv sh\,\mathcal{T}$ and $\mathcal{S}_3(\mathcal{T}) \equiv ch\,\mathcal{T}$, both asymptotic approximations to $\mathcal{S}_1(\mathcal{T}) \equiv e^{\mathcal{T}}$, are derived from RWM.

But incompatibility of $\mathcal{S}_1(\mathcal{T})$ with LT does not entail incompatibility of $\mathcal{S}_1(\mathcal{T})$ with LT', i.e., the differential LT. So we may ask if it is possible to devise a new SS model by means of LT' & LP, applying the method of Milne, instead of basing it on RWM, employing the method of Walker.

We are therefore faced with the choice between $d\widehat{\mathcal{T}}^{\,2} = dt^2 - dr^2$ together with LT', and $d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - e^{2\mathcal{T}} d\sigma^2$ in combination with some other and hitherto unknown transformations. Following Bondi and choosing the standard solution we have not only a cosmic time, but also what Milne called a 'public' 3-space. Making the other choice our 3-space remains 'private' in the sense that it can only be described in the perspective of an observer. But we still have a cosmic time, $\widehat{\mathcal{T}}$.

Hence our problem: Is a cosmology based on LP in the strict sense at all feasible?

1. A simple derivation of LT.

A simple way of obtaining the Lorentz Transformations (LT) of Special Relativity (SR) - not very rigorous, but very illuminating - would begin with the Galileo Transformations (GT), unprimed coordinates referring to an observer O and primed to another observer O', taking O & O' to be in collinear motion with uniform relative velocity v, as reckoned from O to O':

(1.1) $$t' = t\,,\; x' = x - vt\,,\; y' = y\,,\; z' = z$$

GT conform to the relativity principle (RP), but not to that of a constant light speed (LP). Hence, if O coincide with O' at the event $(t_o, x_o, 0, 0) = (t'_o, x'_o, 0, 0) = (0,0,0,0)$, a light wave emerging from that event could not be spherical with respect to both observers at the same time, since equation $c^2t^2 = x^2+y^2+z^2$ is not transformed into the similar equation $c^2t'^2 = x'^2+y'^2+z'^2$. So we cannot put $c \equiv c'$. This shows that neither can we synchronize clocks using reflected light, nor can we interpret spatial distance as light-time measured by reflected radar signals.

If, by contrast, we insist on $c \equiv c'$, in agreement with the apparent results of observation and experiment, we shall have to modify GT accordingly. However, we may preserve the standard convention $v' = -v$, now enlightened by the fact that the numerical value of both velocities are expressible as the same fraction of the speed of light: $|v|/c = |v'|/c$. A further simplification can be obtained if we put $c \equiv c' \equiv 1$, thus interpreting all speeds as fractions of the speed of light. How should GT be modified? The only parameter we dispose of is $|v'| = |v| =$ *const*.

So let us define a new function: $\gamma \equiv \gamma(v)$. According to RP, γ would have to be invariant. The simplest assumption is that γ only affects transformations in the direction of the x-axis leaving the two other spatial dimensions unaffected. But since t is involved in the transformation of x to x', it may affect t' also, thereby necessitating our renunciation of the classical transformation $t = t'$. An additive constant would hardly do the job. Let us try if γ could be a multiplicative factor:

(1.2) $$x' = \gamma(x - vt)\,,\; y' = y\,,\; z' = z\,,\; x = \gamma(x' - v't')$$

In fact, assuming $v' = -v$, it has to be done this way, for v is the velocity of a fix-point in the frame of O' as calculated by O: $dx' = \gamma(dx - vdt) = 0 \;\Rightarrow\; dx = vdt$, just as v' is the velocity of a fix-point in the frame of O as calculated by O': $dx = \gamma(dx' - v'dt') = 0 \;\Rightarrow\; dx' = v'dt'$. From $x' = \gamma(x - vt)$ combined with $x = \gamma(x' + vt')$ we then derive the transformation for time:

$$x = \gamma(x' + vt') = \gamma\{\gamma(x - vt) + vt'\} \;\Rightarrow\; t' = \gamma\{t - x(1 - \gamma^{-2})/v\}$$

In case of light propagation, as we have seen: $t^2 - (x^2+y^2+z^2) = t'^2 - (x'^2+y'^2+z'^2) = 0$. Further, all velocities being fractions of the velocity of light, the following must hold in general:

$$t^2 - (x^2+y^2+z^2) = t'^2 - (x'^2+y'^2+z'^2)$$

On account of $y' = y$ & $z' = z$ we accordingly obtain: $t^2 - x^2 = t'^2 = x'^2$. It is therefore evident that the temporal co-ordinates t & t' must transform in the same way as the spatial ones x & x':

(1.3) $$t' = \gamma\{t - x(1 - \gamma^{-2})/v\} = \gamma(t - vx)$$

From this it is easy to derive a precise expression for the γ-factor; we obtain:

(1.4) $$\gamma = 1/\sqrt{1 - v^2}$$

Hence we shall claim that eqs. $x' = \gamma(x - vt)$ & $x = \gamma(x' + vt')$ contain the very germ of LT.

In the next section we shall consider an important way of interpreting the γ-factor.

2. *The importance of the γ-factor.*

Granted LP and $c \equiv c' \equiv 1$, the natural definition of coordinates is the standard one:

(2.1) $$t \equiv \tfrac{1}{2}(T_3{+}T_1) \Downarrow \; x \equiv \tfrac{1}{2}(T_3{-}T_1)$$
$$dT_3 = dt + dx \;,\; dT_1 = dt - dx$$

(2.1') $$t' \equiv \tfrac{1}{2}(T_4'{+}T_2') \Downarrow \; x' \equiv \tfrac{1}{2}(T_4'{-}T_2')$$
$$dT_4' = dt' + dx' \;,\; dT_2' = dt' - dx'$$

Why capitals? The point is that T_i & T_i' are read directly off the master-clocks of O & O', resp., denoting the proper times of those clocks, while t & t' are read off the slave-clocks fixed to the co-moving frames F_o & $F_{o'}$ of O & O', resp., and denote the standard frame times of F_o & $F_{o'}$. So *master-clocks* show *proper time* T, while *slave clocks* show *frame time* t.

Imagine a zig-zag signal passing to & fro between O & O' directly, without any delay:

$$... < T_1 < T_2' < T_3 < T_4' < ...$$

These are epochs supposed to be read off the master-clocks of O & O' at the events of reflection. Granted RP, T_4' must be the same function of T_3 as T_3 of T_2', and as T_2' of T_1, viz. $T_{i+1} \equiv \psi(T_i)$:

(2.2) $$T_4' = \psi(T_3) \;,\; T_3 = \psi(T_2') \;,\; T_2' = \psi(T_1)$$

The signal-function ψ forms the core of the Milne-Whitrow derivation of LT which was hailed as transcendental by J.R. Lucas [1971]. The relative motion of O & O' being inertial and reciprocal, $v = -v'$, the function ψ must be linear, i.e. of the form $s\,T{+}k$. The Doppler Shift (DS), which we shall identify with its derivative $\psi' = s$, must therefore be constant as well as reciprocal:

(2.3) $$1{+}z \equiv dT_3/dT_2' = s = dT_2'/dT_1 \equiv 1{+}z'$$

(2.4) $$1{+}z = \sqrt{dT_3/dT_1} = \sqrt{\tfrac{dt+dx}{dt-dx}} = \sqrt{\tfrac{1+v}{1-v}} = e^{arth\, v} = s$$

Synchronizing the master-clocks of O & O' to read $T = T' = 0$ by coincidence we must put $k = 0$ at least for signals exchanged directly between O & O'; hence $T_3/T_2' = s = T_2'/T_1$. Further:

(2.5) $$dT_2' = \sqrt{dT_3 dT_1} = \sqrt{(dt{+}dx)(dt{-}dx)} = dt\sqrt{1{-}dx^2/dt^2}$$

(2.5') $$dT_3 = \sqrt{dT_4' dT_2'} = \sqrt{(dt'{+}dx')(dt'{-}dx')} = dt'\sqrt{1{-}dx'^2/dt'^2}$$

Thus, in case of photon transmission: $\frac{dx}{dt} = \frac{dx'}{dt'} = 1$, the element of proper time dT will be zero even though the element of frame time dt is not. Further, for the relative motion of O & O':

(2.6) $$x' = 0 \;\Rightarrow:\; dx = v\,dt \;\Rightarrow\; dt = dT'/\sqrt{1{-}v^2} = \gamma\, dT'$$

(2.6') $$x = 0 \;\Rightarrow:\; dx' = v'dt' \;\Rightarrow\; dt' = dT/\sqrt{1{-}v'^2} = \gamma\, dT$$

This proves that the master-clock of O' showing dT' will appear to be retarded relative to a series of slave-clocks distributed along the x-axis of O, just as the master-clock of O, showing dT, will appear to be retarded relative to a series of slave-clocks distributed along the x'-axis of O'.

So proper time differs from frame time - there is no surprise in this, cf. H. Arzeliés [1966]. What is surprising, however, is that in the case of energy change, due to the performance of work, the retardation will be absolute. Thus a moving clock will lack behind a resting clock of the same construction if it returns to its point of departure after having completed a circuit in space.

Nevertheless, as far as inertial collinear motion is concerned, we can reduce LT to GT for any pair of observers! This will be demonstrated in the following section.

3. *The formal reduction of LT to GT.*

With $\gamma = 1/\sqrt{1-v^2}$, as already shown, the full LT can be stated in the following form:

(3.1) $$x' = \gamma(x-vt)\,,\; y' = y\,,\; z' = z\,,\; x = \gamma(x'+vt')$$

Now insert the values of $t\,\&\,t'$ from $\widehat{T} \equiv t-\frac{x}{v}(1-\gamma^{-1}) \equiv t'-\frac{x'}{v'}(1-\gamma^{-1})$ into LT - lo and behold: we immediately obtain something which is surprisingly similar to the GT of classical physics!

(3.2) $$x' = x - \gamma\, v\, \widehat{T}\,,\; y' = y\,,\; z' = z\,,\; x = x' - \gamma\, v' \widehat{T}$$

But is $\widehat{T}$ a time displayed on physical clocks? Let us take the derivative of $\widehat{T}$ and see what we get:

(3.3) $$d\widehat{T} \equiv dt-\frac{dx}{v}(1-\gamma^{-1}) \equiv dt'-\frac{dx'}{v'}(1-\gamma^{-1})$$

(3.4) $$d\widehat{T}\underset{dx\,=\,v\,dt}{=} dt/\gamma = dT \underset{dx'\,=\,v'dt'}{=} dt'/\gamma = dT'$$

So the clock sought for keeps the same rate as the master-clocks of $O\,\&\,O'$. Further, $x' = x$ for $\widehat{T} = 0$; this shows that the new clock agrees with the two master-clocks at their coincidence. These are the criteria of clock-congruence, or synchrony. Hence $\widehat{T}$ is the common time of $O\,\&\,O'$! That such a time should exist is the great *no*!-*no*! of SR. But "c'est ne pas tout", to quote Poincaré: In fact, as shown by J. Winnie [1970], a simple additive adjustment of time-zero for each single slave clock will suffice to ensure that all the slave-clocks distributed over the entire co-moving frames of $O\,\&\,O'$ will agree by coincidence - and will to continue to do so for ever after!

From the point of view of an observer M situated precisely midway between $O\,\&\,O'$, i.e. $MO \equiv MO'$ - and such a midway observer and his co-moving frame can always be constructed - it is evident not only that the master-clocks of $O\,\&\,O'$ are in perfect synchrony, but also that their co-moving slave-clocks after adjustment of their zeros keep exactly the same rate, $d\widehat{T} = d\hat{t} = d\hat{t}'$. Hence, to make clocks in uniform collinear motion tick in unison is not a question of changing their clock mechanisms, but only a question of adjusting their time-zeros properly! Apparently the trick cannot be performed with more than one pair of frames at a time. But look at this:

Consider the case of three or more observers $O\,\&\,O'\,\&\,O''$ *etc.* in inertial collinear motion. If only they coincide at the same event, $t = t' = t''$, we can always devise an adjustment of time-zeros for their slave-clocks in order to make them all agree, without changing their mechanisms. In fact, we need no more than a single slave-clock in each fix-point of their co-moving frames if only each of these clocks serves as time-keeping mechanism for an unlimited number of pointers, each of these pointers being adjusted with its own time-zero depending on the relative velocity of that observer with respect to whose co-moving slave-clocks it is intended to agree.

Now consider instead a triply infinite (∞^3) set of observer-particles, or particle-observers. Let us assume that the structure of the whole set is defined by the following property: for any non-collinear triple of particle-observers belonging to the set there is a fourth one also member of the set which is the mid-way particle of the first three so that it remains equidistant from those three. My conjecture, then, is that such a set constitutes a substratum of fundamental observers in the sense of Milne, thereby fulfilling his specific formulation of the cosmological principle (CP)!

If this is the case, then all proper distances $\widehat{\mathcal{R}}$ between the members of such a substratum will be subject to the same scale function $\mathcal{S}(\widehat{\mathcal{T}})$, taking the same time $\widehat{\mathcal{T}}$ as its argument. But even though the substratum is everywhere *isotropic* it is a *centroid*, i.e. not homogeneous, cf.§7 & fig.1-2.

4. From "big bang" to "steady state".

As we have seen, Walker rendered the basic 1x1-invariant of Milne's KR thus:

(4.1) $$d\widehat{\mathcal{T}}^{\,2} \;=\; dt^2 - dr^2 \;=\; d\mathcal{T}^2 - \mathcal{T}^2 d\sigma^2$$

This metric he generalized to the standard metric of modern cosmology (RWM), cf. North [1965]:

(4.2) $$d\widehat{\mathcal{T}}^{\,2} \;=\; d\mathcal{T}^2 - \mathcal{S}^2(\mathcal{T})\, d\sigma^2 = \textit{invar.}$$

Here $\mathcal{T}$ is an universal parameter, argument in the scaling function $\mathcal{S}(\mathcal{T})$, and for $d\sigma = 0$ identical to the invariant cosmic time $\widehat{\mathcal{T}}$ which is the common proper time read off the master-clocks of all so-called fundamental observers, members of the substratum. For $d\widehat{\mathcal{T}} = 0$ we obtain:

(4.3) $$\sigma = \int_{\mathcal{T}_1}^{\mathcal{T}_2} \frac{d\mathcal{T}}{\mathcal{S}(\mathcal{T})} = \int_{\mathcal{T}_2}^{\mathcal{T}_3} \frac{d\mathcal{T}}{\mathcal{S}(\mathcal{T})} = \textit{const.}$$

Walker interpreted σ as a co-moving coordinate that characterizes a fundamental observer relative to another fundamental observer, their proper distance being: $\mathcal{R}(\mathcal{T}) = \mathcal{S}(\mathcal{T})\mathcal{R}_o = \mathcal{S}(\mathcal{T})\sigma$. The curvature of 3-space, which is latent in the element $d\sigma$, he left undetermined to a beginning. A new time scale which eliminates the spatial expansion by the factor $\mathcal{S}(\mathcal{T})$, so that the distances between fundamental observers remain constant in time, is definable by: $\tau = \int d\mathcal{T}/\mathcal{S}(\mathcal{T}) + \textit{const.}$ From the constancy of σ we can derive the cosmological red-shift $1+z$ for any standard RWM:

(4.4) $$\frac{d\mathcal{T}_3}{\mathcal{S}(\mathcal{T}_3)} = \frac{d\mathcal{T}_2}{\mathcal{S}(\mathcal{T}_2)} \;\Rightarrow\; \frac{\mathcal{S}(\mathcal{T}_3)}{\mathcal{S}(\mathcal{T}_2)} = \frac{d\mathcal{T}_3}{d\mathcal{T}_2} = 1+z$$

The term "big bang" is due to Gamow, and the term "steady state" to Hoyle. The first real "big bang" model was devised by a catholic priest, abbé Lemaître, who spoke of a primeval atom. Milne instead chose to speak of a transcendent point-event. According to the Friedmann-Lemaître equations of General Relativity (GR), the BB model of Milne and the SS model of Bondi & Gold must both be completely devoid of matter. A tacit presupposition, of course, is the general validity of the field equations of GR. But neither Milne, nor Bondi & Gold, accepted GR.

In fact, KR was meant by Milne to replace both SR & GR. Milne devised his own original and very ingenious theory of gravitation. As I have argued in an earlier paper, Wegener [2000], his idea can be described as "turning Mach's principle upside down" so that, instead of trying (in vain) to explain inertia by reducing it to a kind of gravitation, he proposed to explain gravitation as a kind of inertia by reducing it to local deviations from global (cosmic) symmetry. From this point of view a cosmological model is primarily characterized by its scaling function, and there is therefore no question of a global gravitational field acting as a kind of brake on the universal expansion, hence also not of assuming the universe to be filled up with dark anti-gravitational energy.

The analogy of $ds^2 = dt^2\gamma^{-2} = dt^2 - dr^2$ & $ds^2 = dt^2 - \mathcal{S}^2(t)d\sigma^2$ - no relation to $s \equiv 1+z$ - has been subject to conjecture by G.J. Whitrow [1961], another student of Milne's, who suggested that RWM may be derivable from the strict LP of SR. However, his procedure is not convincing. Moreover, it turns out that some interesting metrics, viz. that of the "steady state" universe of Bondi & Gold and those of two other kindred models, are at variance with his assumption.

These models - as defined by the scale factors: $\mathcal{S}_1(\mathcal{T}) \equiv e^{\mathcal{T}}$, $\mathcal{S}_2(\mathcal{T}) \equiv sh\mathcal{T},\; \mathcal{S}_3(\mathcal{T}) \equiv ch\mathcal{T}$ - present us with a choice between 1) preserving the standard RWM form and discarding LP as a principle of universal validity, or 2) preserving the universal validity of LP and discarding RWM. Instead of following Bondi & Gold by choosing the first option I shall prefer the second one.

5. *Our basis: the strict "light principle".*

We begin by accepting LP as a universal principle for 1-space: $d\widehat{\mathcal{T}}^{\,2} = dt^2 - dr^2 = invar.$ Here we have as yet not decided whether the curvature of 3-space is positive, negative, or zero; but it is well known that the validity of LT, accepted by Milne, presupposes his 3-space to be flat. That a natural choice of the sign of curvature for the three other models hinted at turns out to be negative has no implication for our comparison of them as far as radial motion is concerned.

We shall exploit the insight of §3, the reduction of LT to GT: $r' = \gamma(r - vt) = r - \gamma\, v\, \widehat{\mathcal{T}}$, $\gamma = 1/\sqrt{1-v^2}$; for points fixed to the frame of O' this implies: $dr' = 0 \Rightarrow dr = vdt = \gamma\, v\, d\widehat{\mathcal{T}}$. Let us now assume a dependence of velocity on distance, such as must hold in any SS model:

$$dr_{oo'} \equiv th\, r_{oo'}\, dt_o \equiv sh\, r_{oo'}\, d\widehat{\mathcal{T}} \tag{5.1}$$

$$\gamma = dt_o/d\widehat{\mathcal{T}} = 1/\sqrt{1-v_{oo'}^2} = ch\, r_{oo'} \tag{5.2}$$

Here $r_{oo'}$ is supposed to be the standard light-time distance between two fundamental observers, O & O', just as $v_{oo'} = dr_{oo'}/dt_{oo'} = th\, r_{oo'}$ is supposed to be their (variable) relative velocity.

It follows from the strict LP that the speed of all photons is a universal constant, irrespective of the state of motion of their source; we may thus claim that radar-signals involving the reflection of photons from distant objects measure the same light-time distance out and home, and that the parts of light tracks of reflected photons are additive. Now from the group-properties of LT, together with the general postulate: $v_{ij} = th\, r_{ij}$, follows the group-properties of what I shall call pLT' (pseudo-LT'):

$$dr_{o'} = \gamma_{oo'}(dr_o - th\, r_{oo'} dt_o) = ch\, r_{oo'}\, dr_o - sh\, r_{oo'}\, dt_o \tag{5.3}$$

$$dr_o = \gamma_{o'o}(dr_{o'} - th\, r_{o'o} dt_{o'}) = ch\, r_{o'o}\, dr_{o'} - sh\, r_{o'o}\, dt_{o'} \tag{5.3'}$$

$$dt_{o'} = \gamma_{oo'}(dt_o - th\, r_{oo'} dr_o) = ch\, r_{oo'}\, dt_o - sh\, r_{oo'}\, dr_o \tag{5.4}$$

$$dt_o = \gamma_{o'o}(dt_{o'} - th\, r_{o'o} dr_{o'}) = ch\, r_{o'o}\, dt_{o'} - sh\, r_{o'o}\, dr_{o'} \tag{5.4'}$$

Granted that the 3-space r^3 of light time distance is vectorial: $\vec{r}_{oo'}(\widehat{\mathcal{T}}) + \vec{r}_{o'o''}(\widehat{\mathcal{T}}) + \vec{r}_{o''o}(\widehat{\mathcal{T}}) = 0$ (an assumption that seems to hold good for the model M_1 at least, cf.§7) it is easy to verify that pLT' in themselves (independently of LT') conform to a strict LP: $d\widehat{\mathcal{T}}^{\,2} = dt^2 - dr^2 = dt'^2 - dr'^2 = invar.$ We can therefore conclude that the strict LP may be used as a starting-point for our new SS-models. On the other hand it is obvious that pLT' cannot be integrated to something which is similar to LT. Even so, pLT' enables us to use the method of Milne's KR to formulate a cosmological principle, since the core arguments of Milne to this purpose turn out to be based on LT' rather than on LT.

Before discussing these it should be noticed that a reduction of pLT' to pGT' (pseudo-GT') would presuppose that we consider only the relative motion between pairs of fundamental observers; if we apply pLG' to arbitrary, or accidental, objects different from O & O', we find $dT \neq d\widehat{\mathcal{T}}$:

$$dr_{o'} = ch\, r_{oo'}\, dr_o - sh\, r_{oo'}\, dt_o \equiv dr_o - sh r_{oo'}\, dT_{oo'} \;\Rightarrow \tag{5.6}$$

$$dT_{oo'} = dt_o - dr_o \frac{1-\sqrt{1-th^2 r_{oo'}}}{th\, r_{oo'}} \underset{dr_o = dr_{oo'}}{=} dt_o \sqrt{1-v_{oo'}^2} = dt_o/\gamma = d\widehat{\mathcal{T}} \tag{5.7}$$

$$dr_o = ch\, r_{o'o}\, dr_{o'} - sh\, r_{o'o}\, dt_{o'} \equiv dr_{o'} - sh r_{o'o}\, dT \;\Rightarrow \tag{5.6'}$$

$$dT_{o'o} = dt_{o'} - dr_{o'} \frac{1-\sqrt{1-th^2 r'_{oo'}}}{th\, r'_{oo'}} \underset{dr_{o'} = -dr_{oo'}}{=} dt_{o'} \sqrt{1-v_{o'o}^2} = dt_{o'}/\gamma = d\widehat{\mathcal{T}} \tag{5.7'}$$

So the condition of identifying dT_{ij} with $d\widehat{\mathcal{T}}$ is $v_o = v_{oo'} = th\, r_{oo'}$ from the view point of O and $v_{o'} = v_{o'o} = th\, r_{o'o}$ from the view point of O'. The true, or universal, invariant is: $d\widehat{\mathcal{T}}$.

6. *The cosmological principle of Milne.*

Let us state a few definitions, unprimed entities referring to O and primed ones to O':

(6.1) $$v^2 \equiv v_x^2+v_y^2+v_z^2\ ,\ v'^2 \equiv v_x'^2+v_y'^2+v_z'^2$$

(6.2) $$v \equiv dr/dt\ ,\ v_x \equiv dx/dt\ ,\ v_y \equiv dz/dt\ ,\ v_z \equiv dz/dt$$

(6.2') $$v' \equiv dr'/dt'\ ,\ v_x' \equiv dx'/dt'\ ,\ v_y' \equiv dz'/dt'\ ,\ v_z' \equiv dz'/dt'$$

Milne considered the velocity-distribution of particles in a kinematic substratum as it shows itself to two fundamental observers, O & O', which are themselves members of the substratum. Since O & O' both observe the same set of objects, viz. the substratum, they must agree about:

(6.3) $$f_o(v_x, v_y, v_z)\, dv_x dv_y dv_z = f_{o'}(v_x', v_y', v_z')\, dv_x' dv_y' dv_z'$$

The CP which is assumed to hold for all fundamental observers, members of the substratum, but neither for arbitrary objects, nor for accidental observers not belonging to the substratum, may be understood as a strict or universal RP legitimizing the definability of an invariant cosmic time. Milne himself interpreted his CP as a principle stating the formal identity of the functions f_o & $f_{o'}$:

(6.4) $$\boxed{f_o \equiv f_{o'} \equiv f}$$

In order to investigate the consequences of this identification he introduced the LT':

(6.5) $$dx' = \frac{dx - v_{oo'}dt}{\sqrt{1-v_{oo'}^2}}\ ,\ dy' = dy\ ,\ dz' = dz\ ,\ dt' = \frac{dt - v_{oo'}dx}{\sqrt{1-v_{oo'}^2}}$$

(6.6) $$v_x' = \frac{v_x - v_{oo'}}{1-v_x v_{oo'}}\ ,\ v_y' = \frac{v_y(1-v_{oo'}^2)}{1-v_y v_{oo'}}\ ,\ v_z' = \frac{v_z(1-v_{oo'}^2)}{1-v_z v_{oo'}}$$

Applying partial differentiation to LT' he then derived the following provisional results [1948,§52]:

(6.7) $$f(v_x, v_y, v_z) = f(v_x', v_y', v_z')\frac{\partial v_x' \partial v_y' \partial v_z'}{\partial v_x \partial v_y \partial v_z}$$

(6.8) $$\frac{\partial v_x' \partial v_y' \partial v_z'}{\partial v_x \partial v_y \partial v_z} = \frac{(1-v_{oo'}^2)^2}{(1-v_x v_{oo'})^4}$$

(6.9) $$f(v_x, v_y, v_z) = f\left(\frac{v_x - v_{oo'}}{1-v_x v_{oo'}}, \frac{v_y(1-v_{oo'}^2)}{1-v_y v_{oo'}}, \frac{v_z(1-v_{oo'}^2)}{1-v_z v_{oo'}}\right)\frac{(1-v_{oo'}^2)^2}{(1-v_x v_{oo'})^4}$$

The most general solution of these functional equations Milne [1935] showed to be:

(6.10) $$f(v_x, v_y, v_z)\, dv_x dv_y dv_z = B\,\gamma^4\, dv_x dv_y dv_z$$

(6.11) $$\gamma \equiv \frac{1}{\sqrt{1-v_x^2-v_y^2-v_z^2}}\ ,\ B = \textit{const.}$$

Expressed with polar coordinates, using $d\omega$ to denote a small solid angle, his result can be written:

(6.12) $$\underline{f(v,\omega)\, v^2 dv\, d\omega = B\,\gamma^4\, v^2 dv\, d\omega}$$

In order to pass from this velocity-distribution to the corresponding distribution over 3-space Milne then applied the basic property of his uniformly expanding model, viz. the constancy of the relative velocities between all fundamental observers (members of the subtratum) taken pairwise. At this point, however, we must deviate from his procedure, the relative velocities between pairs of fundamental observers in our SS-model being no longer constant, but increasing with distance. Thus, in order to proceed, we shall exploit the property $\gamma \equiv \frac{1}{\sqrt{1-v^2}} = \frac{1}{\sqrt{1-th^2 r}} = ch\, r$ implied by:

(6.13) $$\underline{d\widehat{\mathcal{T}} = dt/ch\, r = dr/sh\, r = \textit{invar.}}$$

7. The model M_1 of continual creation

Milne makes a very illuminating distinction between the universe as *world-map* and the universe as *world-view* (literally he speaks of 'world-picture', but I prefer the term 'world-view'). Perceived as instantaneous appearance, the universe presents itself to the observer as *world-view*. Defined as simultaneous co-existence, the universe should be *re*-presented as *world-map*.

As *world-map* our SS-model displays exactly the same structure as the BB-model of Milne. As *world-view* the two models appear rather different to the observer. The reason is that an SS-universe is stationary whereas a BB-universe is evolving. Consequently, an SS-universe appears as it is (*w.-view* = *w.-map*) whereas a BB-universe appears differently (*w.-view* $\neq$ *w.-map*).

From LT' (§2) we have: $s = 1{+}z = e^{arth\, v}$. Applying: $v = \frac{dr}{dt} = th\, r\ (\S 6)$, we get:

(7.1) $$s = 1{+}z = e^{arth\, v} = e^{r}$$

It is satisfying to see that S.J. Prokhovnik [1967 app.6] in relation to a uniform expansion model very similar to that of Milne, argues in behalf of the same formula. This yields a natural unit of length:

(7.2) $$s = 1{+}z = e \ \Leftrightarrow\ r = 1$$

Thus, so far, our model fulfils the dimensional postulate of Milne [1948,§72], that no dimensional constants of nature should be allowed to enter the definition of the substratum concept.

According to the midway-particle property as postulated in §3, the substratum must be dense. Considered as an ideal mathematical object the kinematic substratum can therefore be compared to a dynamic reference frame having all distances expanding according to a scale factor $\mathcal{S}(\widehat{\mathcal{T}}) \equiv e^{\widehat{\mathcal{T}}}$. An event, occurring in the substratum and being marked out by a flash - light from that event being perceived by a fundamental observer - need therefore not be located by referring to the co-moving standard frame of that observer, since it can equally well - nay, better, and more direct - be located by reference to that other fundamental observer at which it occurs at the epoch $\widehat{\mathcal{T}}_o$.

The basic equations at the end of §6 are easily integrated, and we just state the result:

(7.3) $$\boxed{\widehat{\mathcal{T}} = t - ln\, ch^2\tfrac{r}{2} = ln[2]th\tfrac{r}{2} - \textit{const.}}$$

Applying the definitions: $\rho \equiv \widehat{\mathcal{R}}/e^{\widehat{\mathcal{T}}} \equiv 2\, th\frac{r}{2}/e^{\widehat{\mathcal{T}}} \equiv \textit{const.}$, calibration of units then implies:

(7.4) $$r = 0 \ \Leftrightarrow\ \widehat{\mathcal{R}} = 0 \ \Leftrightarrow\ \widehat{\mathcal{T}} = t$$

(7.5) $$r = 1 \Leftrightarrow\ \widehat{\mathcal{R}} = 2th\tfrac{1}{2} \ \Leftrightarrow\ t = \widehat{\mathcal{T}} + ln\, ch^2\tfrac{1}{2}$$

If we consider a series of different fundamental particles, we get $d\rho \neq 0$, and, by differentiation:

(7.6) $$e^{\widehat{\mathcal{T}}} d\rho = d\widehat{\mathcal{R}} - \widehat{\mathcal{R}}\, d\widehat{\mathcal{T}}\ ,\ \ d\widehat{\mathcal{T}} = dt - th\tfrac{r}{2}\, dr$$

(7.7) $$dr' \equiv\ e^{t} d\rho = ch\, r\, dr - sh\, r\, dt = dr - sh\, r\, d\widehat{\mathcal{T}}$$

(7.8) $$d\widehat{\mathcal{T}} \underset{d\rho = 0}{=} d\widehat{\mathcal{R}}/\widehat{\mathcal{R}} = dr/sh\, r = dt/ch\, r = dt/\gamma = \textit{invar.}$$

This is precisely what we would expect to characterize a true model of continued creation. Using $v = th\, r = \widehat{\mathcal{R}}/(1{+}\frac{1}{4}\widehat{\mathcal{R}}^2)$ to interpret Milne's formula for velocity-distribution, we finally get:

(7.9) $$\underline{B\, \gamma^4\, v^2 dv\, d\omega\ = B\, sh^2 r\, dr\, d\omega = B\, \widehat{\mathcal{R}}^{\,2} d\widehat{\mathcal{R}}\, d\omega/(1{-}\tfrac{\widehat{\mathcal{R}}^2}{4})^3}$$

Such, then, is the spatial distribution of particles in an SS-substratum respecting the CP of Milne: *the substratum emerges as a centroid having its center everywhere and its circumference nowhere*! Our result is easily translated into *a number-redshift relation* using $r = 2\, arth(\frac{1}{2}\widehat{\mathcal{R}}) = ln(1{+}z)$:

(7.10) $$\underline{N/N_o = 4\pi\, z^2(1{+}\tfrac{1}{2}z)^2 dz/(1{+}z)^3}$$

8. Two asymptotic approximations

The world M_2 is constructed as follows. Accepting LT' - i.e. the differential Lorentz Group - as entailed by the strict LP, we assume a specific velocity-distance relation depending on time:

$M_2.1$ $$\boxed{v = \frac{dr}{dt} = \frac{th\,r}{th\,t} \underset{t\to\infty}{\longrightarrow} th\,r}$$

This is easily integrated, and we obtain the following expressions:

$M_2.2$ $$\int \frac{dr}{th\,r} = \int \frac{dt}{th\,t} + ln\,k \;\Rightarrow\; : \; \frac{sh\,r}{sh\,t} = k \;\Rightarrow .\; \frac{dr}{dt} = \frac{th\,r}{th\,t} = \frac{\sqrt{sh^2r+k^2}}{ch\,r}$$

$M_2.3$ $$\gamma = \frac{1}{\sqrt{1-th^2r/th^2t}} = \frac{1}{\sqrt{1-(sh^2r+k^2)/ch^2r}} = \frac{ch\,r}{\sqrt{1-k^2}}$$

We are now able to express our invariant cosmic time $\widehat{\mathcal{T}}$ as a function of r & k:

$M_2.4$ $$\widehat{\mathcal{T}} + lnk \equiv \int \gamma^{-1} dt = \int \frac{\sqrt{1-k^2}}{\sqrt{sh^2r+k^2}}\, dr \underset{k\,\equiv\,th\rho}{=} \int \frac{1}{\sqrt{ch^2r\,ch^2\rho-1}}\, dr \underset{\rho\,\simeq\,0}{\simeq} ln(2th\tfrac{r}{2}) \equiv ln\widehat{\mathcal{R}}$$

The cosmic red-shift, which for M_2 is a genuine DS, is finally found thus:

$M_2.5$ $$\frac{d\widehat{\mathcal{T}}_3}{d\widehat{\mathcal{T}}} = \frac{dt+dr}{d\widehat{\mathcal{T}}} = \frac{ch\,r+\sqrt{sh^2r+k^2}}{\sqrt{1-k^2}} = \frac{\sqrt{1-k^2}}{ch\,r-\sqrt{sh^2r+k^2}} = \frac{d\widehat{\mathcal{T}}}{dt-dr} = \frac{d\widehat{\mathcal{T}}}{d\widehat{\mathcal{T}}_1}$$

$M_2.6$ $$s = 1+z = \left(\frac{d\widehat{\mathcal{T}}_3}{d\widehat{\mathcal{T}}_1}\right)^{\frac{1}{2}} = \left(\frac{ch\,r+\sqrt{sh^2r+k^2}}{ch\,r-\sqrt{sh^2r+k^2}}\right)^{\frac{1}{2}} = e^{arth\sqrt{th^2r(1-k^2)+k^2}} \underset{k\,\simeq\,0}{\simeq} e^r$$

$M_2.7$ $$1+z = e \Leftrightarrow r = arth\sqrt{\frac{th^2 1-k^2}{1-k^2}} \underset{k\,\simeq\,0}{\simeq} 1$$

The world M_3 is found in a similar way. Accepting LT' - i.e. the differential Lorentz Group - as entailed by the strict LP, we assume another velocity-distance relation also depending on time:

$M_3.1$ $$\boxed{v = \frac{dr}{dt} = \frac{th\,r\,th\,t}{1} \underset{t\to\infty}{\longrightarrow} th\,r}$$

This is just as easily integrated, and we obtain the analogous expressions:

$M_3.2$ $$\int \frac{dr}{th\,r} = \int th\,t\,dt + ln\,k \;\Rightarrow\; : \; \frac{sh\,r}{ch\,t} = k \;\Rightarrow .\; \frac{dr}{dt} = \frac{th\,r\,th\,t}{1} = \frac{\sqrt{sh^2r-k^2}}{ch\,r}$$

$M_3.3$ $$\gamma = \frac{1}{\sqrt{1-th^2r\,th^2t}} = \frac{1}{\sqrt{1-(sh^2r-k^2)/ch^2r}} = \frac{ch\,r}{\sqrt{1+k^2}}$$

We are now able to express our invariant cosmic time $\widehat{\mathcal{T}}$ as another function of r & k:

$M_3.4$ $$\widehat{\mathcal{T}} + lnk \equiv \int \gamma^{-1} dt = \int \frac{\sqrt{1+k^2}}{\sqrt{sh^2r-k^2}}\, dr \underset{k\,\equiv\,tan\rho}{=} \int \frac{1}{\sqrt{(ch^2r\,cos^2\rho-1}}\, dr \underset{\rho\,\simeq\,0}{\simeq} ln(2th\tfrac{r}{2}) \equiv ln\widehat{\mathcal{R}}$$

The cosmic red-shift, which for M_3 is a genuine DS, is finally found thus:

$M_3.5$ $$\frac{d\widehat{\mathcal{T}}_3}{d\widehat{\mathcal{T}}} = \frac{dt+dr}{d\widehat{\mathcal{T}}} = \frac{ch\,r+\sqrt{sh^2r-k^2}}{\sqrt{1+k^2}} = \frac{\sqrt{1+k^2}}{ch\,r-\sqrt{sh^2r-k^2}} = \frac{d\widehat{\mathcal{T}}}{dt-dr} = \frac{d\widehat{\mathcal{T}}}{d\widehat{\mathcal{T}}_1}$$

$M_3.6$ $$1+z = \left(\frac{d\widehat{\mathcal{T}}_3}{d\widehat{\mathcal{T}}_1}\right)^{\frac{1}{2}} = \left(\frac{ch\,r+\sqrt{sh^2r-k^2}}{ch\,r-\sqrt{sh^2r-k^2}}\right)^{\frac{1}{2}} = e^{arth\sqrt{th^2r(1+k^2)-k^2}} \underset{k\,\simeq\,0}{\simeq} e^r$$

$M_3.7$ $$1+z = e \;\Leftrightarrow\; r = arth\sqrt{\frac{th^2 1+k^2}{1+k^2}} \underset{k\,\simeq\,0}{\simeq} 1$$

Both models $MM_{2\text{-}3}$ yield: $k \simeq 0 \Rightarrow \widehat{\mathcal{R}} = e^{\widehat{\mathcal{T}}} k$. But whereas the *hard blow* model M_2 for $k \equiv th\rho$ approximates the *steady state* model M_1 from something like a *big bang*, the *soft flow* model M_3 for $k \equiv tan\rho$ approximates M_1 from something like (the breaking of) a *cosmic egg*!

9. *The γ-factor in disguise*

The kinematic substratum servers as a cosmic "compass of inertia" (K. Gödel) by defining the states of rest and motion in the universe. Whereas fundamental observers may be considered to be locally at rest, all other particles not members of the substratum - let us call them accidental - are distinguished by their motion. Now the substratum is dense, cf. the midway property of §3. Given that an accidental particle A is passing a fundamental observer O with velocity $\vec{v}_{oa} \equiv \vec{v}_{oo'}$ at an instant t_o, we have by the same token also found that other fundamental observer O' relative to which it is instantaneously at rest, their distance being $\vec{r}_{ao'} \equiv \vec{r}_{oo'}$ at the same instant t_o.

So the instantaneous state of motion of an accidental particle is completely specified by two fundamental observers: that with which it coincides, and that with respect to which it is at rest. Now, according to RP, all fundamental particles are equivalent. Granted that the energy in any volume of fixed size, $\pi(sh2r_o - 2r_o)$, is constant, cf. the "energy principle" (PCE), and assuming the equivalence of classical gravitational potential to velocity of escape, $v_\infty^2 = -2\varphi = Gm_o/r$, what appears to be the kinetic energy of A relative to O, $\boldsymbol{T}_{oa} = m_a\gamma_{\vec{v}} = m_a/\sqrt{1-\vec{v}^2}$, must equivalently appear to be the dynamic (potential) energy of A relative to O', $-\boldsymbol{V}_{o'a} = m_a\gamma_{\vec{\varphi}} = m_a/\sqrt{1+2\vec{\varphi}}$. So we shall take $\gamma_{\boldsymbol{v}} - \gamma_{\boldsymbol{\varphi}}$ to be a basic constant for fundamental observers in accordance with:

(9.1) $$\boldsymbol{T} = m_o[c^2](\gamma_{\vec{v}} - 1) = m_{\vec{v}} - m_o$$

(9.2) $$-\boldsymbol{V} = m_o[c^2](\gamma_{\vec{\varphi}} - 1) = m_{\vec{\varphi}} - m_o$$

$$d\boldsymbol{T} = \boldsymbol{F}dr = (\frac{dp}{dt})dr = \frac{d}{dt}(m_o\gamma_v v)dr$$
$$= m_o\{v\frac{d\gamma_v}{dt} + \gamma_v\frac{dv}{dt}\}dr = m_o\{v^2 d\gamma_v + \gamma_v v dv\}$$
$$= m_o\{v^2 d\gamma_v + \gamma_v^{-2} d\gamma_v\} = m_o[c^2]d\gamma_v = dm_{\vec{v}}$$
$$-d\boldsymbol{V} = \boldsymbol{F}dr = \{\frac{d}{dr}(m_o\gamma_\varphi)\}dr = m_o[c^2]d\gamma_\varphi = dm_{\vec{\varphi}}$$

(9.3) $$\underline{\boldsymbol{H} \equiv \boldsymbol{T} + \boldsymbol{V} = m_{\vec{v}} - m_{\vec{\varphi}} = m_o(\gamma_{\vec{v}} - \gamma_{\vec{\varphi}}) = const.}$$

(9.4) $$\underline{\boldsymbol{L} \equiv \boldsymbol{T} - \boldsymbol{V} = m_{\vec{v}} + m_{\vec{\varphi}} - 2m_o = m_o(\gamma_{\vec{v}} + \gamma_{\vec{\varphi}} - 2)}$$

Further, assuming also the principle of least action (PLA), and using the above Lagrangian, we are lead to a variational principle describing the observed perihelion displacement of Mercury:

(9.5) $$\underline{\delta\int_{t_1}^{t_2} \boldsymbol{L}\,dt = \delta\int_{t_1}^{t_2}(m_{\vec{v}} + m_{\vec{\varphi}})dt = 0}$$
$$\Rightarrow \frac{d}{dt}\left(\frac{\partial m_{\vec{v}}}{\partial \dot{q}_i}\right) - \frac{\partial m_{\vec{\varphi}}}{\partial q_i} = 0$$

Following Prokhovnik [1988], an unit-rod moving in an aetherial substratum is reduced by:

(9.6) $$c_{v,\theta}^{-1} = \frac{1}{2}(c_{\rightarrow}^{-1} + c_{\leftarrow}^{-1}) = \frac{\sqrt{1-v^2 sin^2\theta}}{1-v^2}$$

Hence, due to the local asymmetry introduced by the motion, the longitudinal speed of a photon will appear to be $c_{v,0} = 1-v^2$, its transversal speed becoming $c_{v,\frac{\pi}{2}} = \sqrt{1-v^2}$. By analogy we also assume:

(9.7) $$c_{v,\theta}^{-1} = \frac{\sqrt{1-v^2 sin^2\theta}}{1-v^2} \Rightarrow c_{\varphi,\theta}^{-1} = \frac{\sqrt{1+2\varphi sin^2\theta}}{1+2\varphi}$$

Now $\delta\int_{t_1}^{t_2}\frac{2r}{c_\varphi}dt$ for $\theta \simeq 0$ yields the observed delay of radar-signals reflected from a planet, while the observed deflection of light rays near a massive body is found by a Fermat principle:

(9.8) $$\delta\int_{t_1}^{t_2}\frac{dr}{c_\varphi} = \delta\int_{r_1}^{r_2}\frac{\sqrt{1+2\varphi sin^2\theta}}{1+2\varphi}dr = 0$$

O. Conclusion

Einstein initiated his remarkable career by doing away with classical absolute simultaneity. Although the idea of a cosmic time emerged as a natural development of his GR, it seems true to say that its viability is in deep conflict with the very spirit of 'Einsteinianism' according to which it is "characteristic of thought in physics, as of thought in natural science generally, that it endeavours in principle to make do with 'space-like' concepts alone", Einstein [1954].

Now a cosmic time $\widehat{\mathcal{T}}$ is definable not only for all models based on RWM, but for all models conformal to the strict LP and to the CP of Milne. It is therefore hardly possible to eliminate time. So the prospect of completing "Einstein's revolution", the elimination of time from natural science, seems as bad as ever. Maybe we can hope for an end to all loose talking about "the end of time"? This would be relieving, since the problems of space are no less pertinent.

The proper space of models based on RWM, e.g., allows expansion with superluminal speed. With models based on the strict LP, the speed of a material body can never exceed the speed of light. Most students of relativity feel comfortable with space derived from light-time distance; but are they equally comfortable with a "real" contraction of bodies in the direction of motion?

Our $MM_{1\text{-}3}$ concept of proper distance, $\widehat{\mathcal{R}} \equiv 2\,th\frac{r}{2} \to e^{\widehat{\mathcal{T}}}k$, may look strange to most people. Nevertheless, it has a simple explanation, being the distance between two fundamental observers, O & O', as defined in the frame of a collinear midway-particle, M. Imagine $v_{om} = th\,r_{om} = th\frac{1}{2}r_{oo'}$ and $v_{o'm} = th\,r_{o'm} = th\frac{1}{2}r_{o'o}$ both approaching 1 as $r \to \infty$; then, just as $v_{om} - v_{o'm} \underset{r\to\infty}{\longrightarrow} 2$, we have:

$$\widehat{\mathcal{R}}_{oo'} = 2\,th\tfrac{1}{2}r_{oo'} = thr_{om} - thr_{o'm} \underset{r\to\infty}{\longrightarrow} 2 \tag{O.1}$$

So, whereas light-time distance r is simply additive, proper distance $\widehat{\mathcal{R}}$ adds up like $2\,th\frac{r}{2}$. This means that $MM_{1\text{-}3}$ display a density increasing with distance not only according to world-view, but also according to world-map, i.e., $MM_{1\text{-}3}$ can only be perceived and described in a perspective. In this way our world-models conform to the description by Nicholas of Cusa [ca.1450]:

The world machinery is as if it had its center everywhere and its circumference nowhere, its circumference and center being no other than God who is both everywhere and nowhere.

From every possible point of view, defined by reference to any single fundamental observer, the universe can be interpreted as a steady stream, or wave, confined within the limits of a sphere, which may be standing (M_1, or "steady state"), expanding from a transcendent point-event (M_2, or "hard blow"), or expanding from the breaking of "a cosmic egg" (M_3, or "soft flow").

What is surprising is that a receding particle will reach the border of the universe at $r \simeq \infty$, or $\widehat{\mathcal{R}} \simeq 2$, within a finite time, in M_1: $\widehat{\mathcal{T}} = ln\frac{2}{k}$. We may say that, at this very instant, the particle leaves the universe, it does no longer exist. Thus everything which can properly said to exist does so within the world sphere. The energy of this sphere must be constant - zero, say.

With our world M_1, we are therefore confronted with the picture of an eternal universe where the energy lost at the periphery is compensated by a steady gain in energy at the center (everywhere), from which a steady ethereal stream of matter and light is pouring out towards all possible directions. And MM_{2-3} are just two different approximations to the same overall view.

For this reason our new "Milne" models $MM_{1\text{-}3}$ can be claimed to represent a true synthesis of the apparently conflicting philosophical views of Parmenides and Herakleitos.

Figure 1. A snapshot of the Milne model M_0, a pseudo-sphere expanding with the speed of light

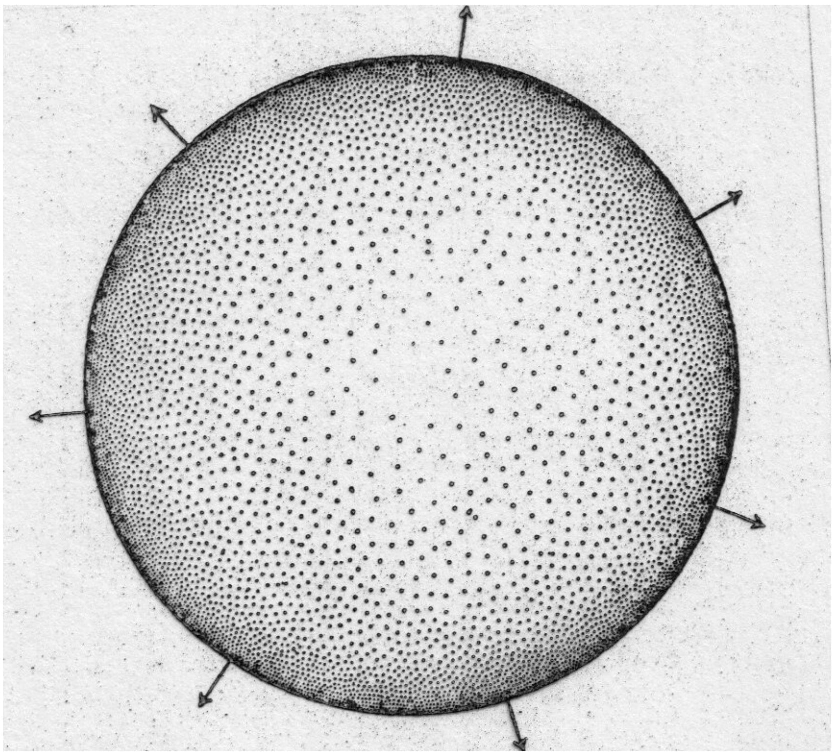
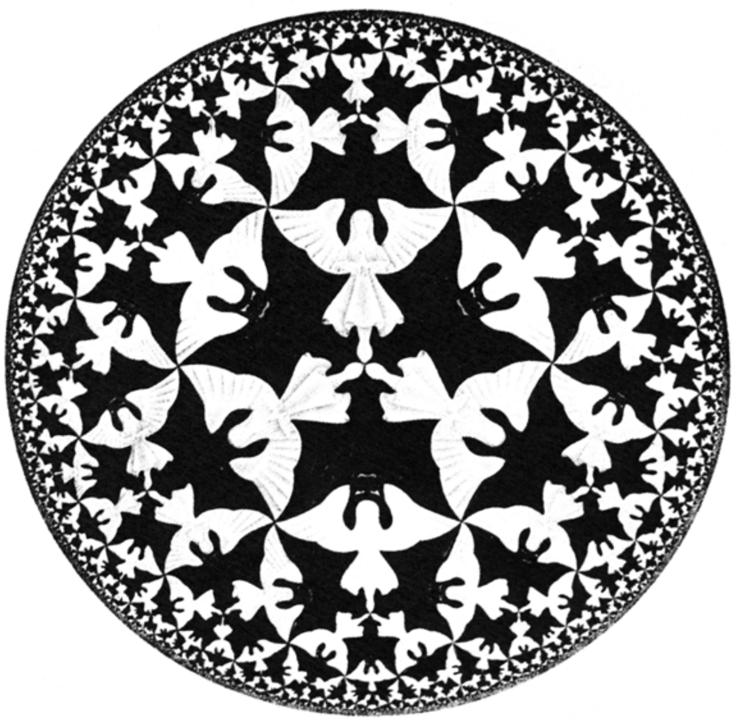


Figure 2. M.C. Escher: 'Circle Limit 4', compare $WW_{1\text{-}3}$ according to 'world-view' (cf.app.1)



Appendix 1: The LP-metrics of MM$_{1\text{-}3}$

The common world-map of worlds MM$_{1\text{-}3}$: a hyperboloid of infinite 3-space,
unfolds M$_i$ as it is in itself: a transcendent unity of simultaneous co-existence

$$\underline{d\widehat{\mathcal{T}}^{\,2} = dt^2 - dr^2 - sh^2 r\,(d\theta^2 + sin^2\theta d\phi^2) = \textit{invariant}}$$

The common world-view of worlds MM$_{1\text{-}3}$: a pseudo-sphere of finite 3-space,
depicts M$_i$ as it appears to us: an observable reality of shells of increasing age

$$\underline{dt^2 = d\widehat{\mathcal{T}}^{\,2} + \{d\widehat{\mathcal{R}}^{\,2} + \widehat{\mathcal{R}}^{\,2}(d\theta^2 + sin^2\theta\, d\phi^2)\}(1 - \tfrac{1}{4}\widehat{\mathcal{R}}^{\,2})^{-2} = \textit{stationary}}$$

Appendix 2: pGT' have no group properties

$$\underline{dr_{o'} = dr_o - sh\, r_{oo'}\, d\widehat{\mathcal{T}}} \;\Leftrightarrow\; \underline{dr_o = dr_{o'} - sh\, r_{o'o}\, d\widehat{\mathcal{T}}}$$
$$dr_{o''} = dr_{o'} - sh\, r_{o'o''}\, d\widehat{\mathcal{T}} \;\Leftrightarrow\; dr_{o'} = dr_{o''} - sh\, r_{o'o''}\, d\widehat{\mathcal{T}}$$

$$\therefore\; \underline{dr_{o''} = dr_{o'} - sh\, r_{o'o''}\, d\widehat{\mathcal{T}} = dr_o - (sh\, r_{oo'} + sh\, r_{o'o''})\, d\widehat{\mathcal{T}} \neq dr_o - sh\, r_{oo''}\, d\widehat{\mathcal{T}}}$$

$$\widehat{\mathcal{T}} = t_o - ln\, ch^2(\tfrac{1}{2} r_{oo'}) = t_{o'} - ln\, ch^2(\tfrac{1}{2} r_{o'o})$$
$$d\widehat{\mathcal{T}} = dt_o - th(\tfrac{1}{2} r_{oo'})\, dr_{oo'} = dt_{o'} - th(\tfrac{1}{2} r_{o'o})\, dr_{o'o}$$
$$th(\tfrac{1}{2} r_{oo'}) \neq th(\tfrac{1}{2} r_o) \;\wedge\; th(\tfrac{1}{2} r_{o'o}) \neq th(\tfrac{1}{2} r_{o'})$$

Appendix 3: pLT' possess group properties

$$\underline{dr_{o'} = ch\, r_{oo'}\, dr_o - sh\, r_{oo'}\, dt_o} \;\Downarrow\; \underline{dr_o = ch\, r_{o'o}\, dr_{o'} - sh\, r_{o'o}\, dt_{o'}}$$
$$dt_{o'} = ch\, r_{oo'}\, dt_o - sh\, r_{oo'}\, dr_o \;\wedge\; dt_o = ch\, r_{o'o}\, dt_{o'} + sh\, r_{o'o}\, dr_{o'}$$

$$\underline{dr_{o''} = ch\, r_{o'o''}\, dr_{o'} - sh\, r_{o'o''}\, dt_{o'}} \;\Downarrow\; \underline{dr_{o'} = ch\, r_{o''o'}\, dr_{o''} - sh\, r_{o''o'}\, dt_{o''}}$$
$$dt_{o''} = ch\, r_{o'o''}\, dt_{o'} - sh\, r_{o'o''}\, dr_{o'} \;\wedge\; dt_{o'} = ch\, r_{o''o'}\, dt_{o''} + sh\, r_{o''o'}\, dr_{o''}$$

$$dr_{o''} = ch\, r_{o'o''}\, dr_{o'} - sh\, r_{o'o''}\, dt_{o'} =$$
$$= ch\, r_{o'o''}(ch\, r_{oo'}\, dr_o - sh\, r_{oo'}\, t_o) - sh\, r_{o'o''}(ch\, r_{oo'}\, dt_o - sh\, r_{oo'}\, dr_o)$$
$$= ch\, r_{o'o''} ch\, r_{oo'}\, dr_o - ch\, r_{o'o''} sh\, r_{oo'}\, dt_o - sh\, r_{o'o''} ch\, r_{oo'}\, dt_o + sh\, r_{o'o''} sh\, r_{oo'}\, dr_o$$
$$= (ch\, r_{o'o''} ch\, r_{oo'} + sh\, r_{o'o''} sh\, r_{oo'})\, dr_o - (ch\, r_{o'o''} sh\, r_{oo'} + sh\, r_{o'o''} ch\, r_{oo'})\, dt_o$$
$$= ch(r_{o'o''} + r_{oo'})\, dr_o - sh(r_{o'o''} + r_{oo'})\, dt_o = ch\, r_{oo''}\, dr_o - sh\, r_{oo''}\, dt_o$$

$$\therefore \underline{dr_{o''} = ch\, r_{o'o''}\, dr_{o'} - sh\, r_{o'o''}\, dt_{o'} = ch\, r_{oo''}\, dr_o - sh\, r_{oo''}\, dt_o}$$
$$dr_{o''} = 0 \;\Rightarrow\; : dr_{o'} = dr_{o'o''} \wedge dr_o = dr_{oo''} \;\Rightarrow .$$
$$v_{o'o''} = \tfrac{dr_{o'o''}}{dt_{o'}} = th\, r_{o'o''} \wedge v_{oo''} = \tfrac{dr_{oo''}}{dt_o} = th\, r_{oo''}$$

$$dt_{o''} = ch\, r_{o'o''}\, dt_{o'} - sh\, r_{o'o''}\, dr_{o'} =$$
$$= ch\, r_{o'o''}(ch\, r_{oo'}\, dt_o - sh\, r_{oo'}\, dr_o) - sh\, r_{o'o''}(ch\, r_{oo'}\, dr_o - sh\, r_{oo'}\, dt_o)$$
$$= ch\, r_{o'o''} ch\, r_{oo'}\, dt_o - ch\, r_{o'o''} sh\, r_{oo'}\, dr_o - sh\, r_{o'o''} ch\, r_{oo'}\, dr_o + sh\, r_{o'o''} sh\, r_{oo'}\, dt_o$$
$$= (ch\, r_{o'o''} ch\, r_{oo'} + sh\, r_{o'o''} sh\, r_{oo'})\, dt_o - (ch\, r_{o'o''} sh\, r_{oo'} + sh\, r_{o'o''} ch\, r_{oo'})\, dr_o$$
$$= ch(r_{o'o''} + r_{oo'})\, dt_o - sh(r_{o'o''} + r_{oo'})\, dr_o = ch\, r_{oo''}\, dt_o - sh\, r_{oo''}\, dr_o$$

$$\therefore \underline{dt_{o''} = ch\, r_{o'o''}\, dt_{o'} - sh\, r_{o'o''}\, dr_{o'} = ch\, r_{oo''}\, dt_o - sh\, r_{oo''}\, dr_o}$$
$$dr_{o''} = 0 \;\Rightarrow\; : d\widehat{\mathcal{T}} = dt_{o'} = dt_{o''}/ch\, r_{o'o''} = dt_o/ch\, r_{oo''}$$

Appendix 4: $WW_{0\text{-}3}$ as based on RWM

The "Walker" worlds $WW_{0\text{-}3}$ are presented in the usual way as derived from their scale functions by means of the RWM, i.e., in a form incompatible with LT' and the strict version of LP.

Milne's "big bang" model M_0 (Walker's model W_0)

$$\tau \equiv \int \frac{d\mathcal{T}}{\mathcal{S}_0(\mathcal{T})} + C \equiv \int \frac{d\mathcal{T}}{\mathcal{T}} + C$$

$$C = 1 \Rightarrow: \ \tau = ln\mathcal{T} + 1 \Rightarrow . \ \tau = \left\{\begin{smallmatrix}\infty\\1\\-\infty\end{smallmatrix}\right. \Leftrightarrow \mathcal{T} = \left\{\begin{smallmatrix}\infty\\1\\0\end{smallmatrix}\right.$$

$$\underline{d\rho \propto d\tau \ \Rightarrow \ \rho = [ln\mathcal{T}]_{\mathcal{T}_1}^{\mathcal{T}_2} = [ln\mathcal{T}]_{\mathcal{T}_2}^{\mathcal{T}_3} = \text{const.}}$$

$$t = \tfrac{1}{2}(\mathcal{T}_3 + \mathcal{T}_1) = \mathcal{T}\, ch\rho \ \Rightarrow \ dt = d\mathcal{T}\, ch\rho + \mathcal{T}\, d\rho\, sh\rho$$

$$r = \tfrac{1}{2}(\mathcal{T}_3 - \mathcal{T}_1) = \mathcal{T}\, sh\rho \ \Rightarrow \ dr = d\mathcal{T}\, sh\rho + \mathcal{T}\, d\rho\, ch\rho$$

$$\frac{dr}{dt} \underset{d\rho = 0}{=} \frac{r}{t} = th\rho \ \Rightarrow \ \gamma \equiv \frac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho = 0}{=} \frac{1}{\sqrt{1 - th^2\rho}} = ch\rho$$

$$\underline{d\widehat{\mathcal{T}}^{\,2} \equiv dt^2 - dr^2 - r^2(d\theta^2 + sin^2\theta\, d\phi^2) =}$$

$$\underline{= d\mathcal{T}^2 - \mathcal{T}^2\{d\rho^2 + sh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\} =}$$

$$\underline{= e^{2\tau}\{d\tau^2 - d\rho^2 - sh^2\rho(d\theta^2 + sin^2\theta\, d\phi^2)\}}$$

The original "steady state" model W_1 of Bondi & Gold

$$\tau \equiv \int \frac{d\mathcal{T}}{\mathcal{S}_1(\mathcal{T})} + C \equiv \int \frac{d\mathcal{T}}{exp\mathcal{T}} + C = -e^{-\mathcal{T}} + C$$

$$C = 1 \Rightarrow: \ e^{\mathcal{T}} = \frac{1}{1-\tau} \Rightarrow . \ \tau = \left\{\begin{smallmatrix}1\\0\\-\infty\end{smallmatrix}\right. \Leftrightarrow \mathcal{T} = \left\{\begin{smallmatrix}\infty\\0\\-\infty\end{smallmatrix}\right.$$

$$\underline{d\rho \propto d\tau \ \Rightarrow \ \rho = [-e^{-\mathcal{T}}]_{\mathcal{T}_1}^{\mathcal{T}_2} = [-e^{-\mathcal{T}}]_{\mathcal{T}_2}^{\mathcal{T}_3} = \text{const.}}$$

$$\rho = e^{-\mathcal{T}_1} - e^{-\mathcal{T}} = e^{-\mathcal{T}} - e^{-\mathcal{T}_3} \ \Rightarrow \ e^t\rho = e^r - e^{t-\mathcal{T}} = e^{t-\mathcal{T}} - e^{-r}$$

$$e^{t-\mathcal{T}} = ch\, r \ \Rightarrow \ d\mathcal{T} = dt - dr\, th\, r \quad , \quad e^t\rho = sh\, r \ \Rightarrow \ e^{\mathcal{T}} d\rho = dr - dt\, th\, r$$

$$d\mathcal{T}^2 - e^{2\mathcal{T}} d\rho^2 = (dt^2 - dr^2)(1 - th^2 r) = (dt^2 - dr^2)ch^{-2}r$$

$$\underline{d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - e^{2\mathcal{T}}\{d\rho^2 + \rho^2(d\theta^2 + sin^2\theta\, d\phi^2)\} =}$$

$$\underline{= \{dt^2 - dr^2 - sh\, r^2(d\theta^2 + sin^2\theta\, d\phi^2)\}ch^{-2}r}$$

$$\underline{\frac{dr}{dt}} \underset{d\rho = 0}{=} \underline{th\, r} \ \Rightarrow \ \gamma \equiv \frac{1}{\sqrt{1 - dr^2/dt^2}} \underset{d\rho = 0}{=} \frac{1}{\sqrt{1 - th^2 r}} = ch\, r$$

The "hard blow" model W_2 ($\neq$ "hard blow"model M_2)

$$\tau \equiv \int \frac{d\mathcal{T}}{\mathcal{S}_2(\mathcal{T})} + C \equiv \int \frac{d\mathcal{T}}{sh\mathcal{T}} + C = ln\, th\tfrac{1}{2}\mathcal{T} + C$$

$$C = 1 - ln\, th\tfrac{1}{2} \Rightarrow: \ th\tfrac{1}{2}\mathcal{T} = e^{\tau - 1 + lnth\frac{1}{2}} \Rightarrow . \ \tau = \left\{\begin{smallmatrix}1-lnth\frac{1}{2}\\1\\-\infty\end{smallmatrix}\right. \Leftrightarrow \mathcal{T} = \left\{\begin{smallmatrix}\infty\\1\\0\end{smallmatrix}\right.$$

$$\underline{d\rho \propto d\tau \ \Rightarrow \ \rho = [ln\, th\tfrac{1}{2}\mathcal{T}]_{\mathcal{T}_1}^{\mathcal{T}_2} = [ln\, th\tfrac{1}{2}\mathcal{T}]_{\mathcal{T}_2}^{\mathcal{T}_3} = \text{const.}}$$

$$e^\rho = th\tfrac{1}{2}\mathcal{T}/th\tfrac{1}{2}\mathcal{T}_1 = th\tfrac{1}{2}\mathcal{T}_3/th\tfrac{1}{2}\mathcal{T}_2$$

$$t = \tfrac{1}{2}(\mathcal{T}_3 + \mathcal{T}_1) = arth(th\tfrac{1}{2}\mathcal{T}e^\rho) + arth(th\tfrac{1}{2}\mathcal{T}e^{-\rho}) = arth(th\mathcal{T}\, ch\rho)$$

$$r = \tfrac{1}{2}(\mathcal{T}_3 - \mathcal{T}_1) = arth(th\tfrac{1}{2}\mathcal{T}e^\rho) - arth(th\tfrac{1}{2}\mathcal{T}e^{-\rho}) = arth(sh\mathcal{T}\, sh\rho)$$

$$r = arth(sh\mathcal{T}\,sh\rho) \Rightarrow dr = \frac{ch\mathcal{T}\,d\mathcal{T}\,sh\rho + sh\mathcal{T}\,ch\rho\,d\rho}{1-sh^2\mathcal{T}\,sh^2\rho}$$

$$t = arth(th\mathcal{T}\,ch\rho) \Rightarrow dt = \frac{ch^{-2}\mathcal{T}\,d\mathcal{T}\,ch\rho + th\mathcal{T}\,sh\rho\,d\rho}{1-th^2\mathcal{T}\,ch^2\rho} = \frac{d\mathcal{T}\,ch\rho + sh\mathcal{T}\,ch\mathcal{T}\,sh\rho\,d\rho}{1-sh^2\mathcal{T}\,sh^2\rho}$$

$$\underline{d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - sh^2\mathcal{T}\{d\rho^2 + sh^2\rho(d\theta^2 + sin^2\theta\,d\phi^2)\} =}$$

$$\underline{= \{dt^2 - dr^2 - sh^2 r(d\theta^2 + sin^2\theta\,d\phi^2)\}ch^{-2}r}$$

$$\underline{\frac{dr}{dt}}\ \underset{d\rho=0}{\equiv}\ \underline{\frac{th\,r}{th\,t}} \Rightarrow \gamma \equiv \frac{1}{\sqrt{1-dr^2/dt^2}}\ \underset{d\rho=0}{\equiv}\ \frac{1}{\sqrt{1-th^2r/th^2t}}$$

The "soft flow" model W_3 ($\neq$ "soft flow"model M_2)

$$\tau \equiv \int \frac{d\mathcal{T}}{\mathcal{S}_3(\mathcal{T})} + C \equiv \int \frac{d\mathcal{T}}{ch\mathcal{T}} + C = arsin\,th\mathcal{T} + C$$

$$C = 0 \Rightarrow :\ th\mathcal{T} = sin\tau \Rightarrow .\ \tau = \left\{ \begin{matrix} \pi/2 \\ 0 \\ (-\pi/2) \end{matrix} \right. \Leftrightarrow \mathcal{T} = \left\{ \begin{matrix} \infty \\ 0 \\ (-\infty) \end{matrix} \right.$$

$$\underline{d\rho \propto d\tau \Rightarrow \rho = [arsin\,th\mathcal{T}]_{\mathcal{T}_1}^{\mathcal{T}_2} = [arsin\,th\mathcal{T}]_{\mathcal{T}_2}^{\mathcal{T}_3} = const.}$$

$$th\mathcal{T}_3 = sin(arsin\,th\mathcal{T} + \rho)\ ,\ th\mathcal{T}_1 = sin(arsin\,th\mathcal{T} - \rho)$$

$$th2t = th(\mathcal{T}_3 + \mathcal{T}_1) = \frac{2\,th\mathcal{T}\,cos\rho}{cos^2\rho + th^2\mathcal{T}} \Rightarrow :th\,t = th\mathcal{T}/cos\rho \Rightarrow dt = \frac{d\mathcal{T}\,cos\rho + sh\mathcal{T}\,ch\mathcal{T}\,sin\rho d\rho}{1-ch^2\mathcal{T}\,sin^2\rho}$$

$$th2r = th(\mathcal{T}_3 - \mathcal{T}_1) = \frac{2\sqrt{1-th^2\mathcal{T}}\,sin\rho}{1-th^2\mathcal{T} + sin^2\rho} \Rightarrow :th\,r = ch\mathcal{T}\,sin\rho \Rightarrow dr = \frac{sh\mathcal{T}\,d\mathcal{T}\,sin\rho + ch\mathcal{T}\,cos\rho d\rho}{1-ch^2\mathcal{T}\,sin^2\rho}$$

$$\underline{d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - ch^2\mathcal{T}\{d\rho^2 + sin^2\rho(d\theta^2 + sin^2\theta\,d\phi^2)\} =}$$

$$\underline{= \{dt^2 - dr^2 - sh^2 r(d\theta^2 + sin^2\theta\,d\phi^2)\}ch^{-2}r}$$

$$\underline{\frac{dr}{dt}}\ \underset{d\rho=0}{\equiv}\ \underline{\frac{th\,r\,th\,t}{1}} \Rightarrow \gamma \equiv \frac{1}{\sqrt{1-dr^2/dt^2}}\ \underset{d\rho=0}{\equiv}\ \frac{1}{\sqrt{1-th^2r\,th^2t}}$$

The three "Walker"-models $WW_{1\text{-}3}$

$$\mathcal{R}_{AB}(\mathcal{T}) \equiv \mathcal{S}_i(\mathcal{T})\rho_{AB}\ ,\ \tau \equiv \int d\mathcal{T}/\mathcal{S}_i(\mathcal{T}) + C$$

$$\rho = \int_{\mathcal{T}_1}^{\mathcal{T}_2} \frac{d\mathcal{T}}{\mathcal{S}_i(\mathcal{T})} = \int_{\mathcal{T}_2}^{\mathcal{T}_3} \frac{d\mathcal{T}}{\mathcal{S}_i(\mathcal{T})} = const. \Rightarrow .\ \frac{d\mathcal{T}_3}{\mathcal{S}_i(\mathcal{T}_3)} = \frac{d\mathcal{T}_2}{\mathcal{S}_i(\mathcal{T}_2)} \Rightarrow \frac{\mathcal{S}_i(\mathcal{T}_3)}{\mathcal{S}_i(\mathcal{T}_2)} = \frac{d\mathcal{T}_3}{d\mathcal{T}_2} = 1 + z$$

$$\underline{d\widehat{\mathcal{T}}^{\,2} = d\mathcal{T}^2 - \mathcal{S}_i(\mathcal{T})\{d\rho^2 + f^2(\rho)(d\theta^2 + sin^2\theta\,d\phi^2)\} =}$$

$$\underline{= \{dt^2 - dr^2 - sh\,r^2(d\theta^2 + sin^2\theta\,d\phi^2)\}ch^{-2}r}$$

$$d\widehat{\mathcal{T}}\ \underset{d\theta=d\phi=0}{=}\ \sqrt{dt^2 - dr^2}/ch\,r = dt/\gamma\,ch\,r\ \underset{r\simeq 0}{\simeq}\ dt/\gamma$$

$$\widehat{\mathcal{T}} = \mathcal{T} \Leftrightarrow d\rho = d\theta = d\phi = 0\ ,\ \widehat{\mathcal{T}} = t \Leftrightarrow r = dr = 0$$

A final remark

The difference between our "Milne" models $MM_{1\text{-}3}$ and our "Walker" models $WW_{1\text{-}3}$ is this: For M_1, world-view is identical to world-map in the sense that we see the visible objects as they were when they emitted their light, and their general structure then is similar to their general structure now. The observable universe is potentially infinite, actual infinity reigns at the periphery, nothing remains "outside" beyond a horizon. For W_1, this is not the case: the observable universe, being finite, is just an infinitesimal drop in an infinite ocean of real existence, so that there is an impenetrable horizon separating what what we can see from what there was/is. Philosophically, this is not satisfying since it makes it impossible to state a sensible definition of the total amount of energy comprised by W_1. Finally, what applies to M_1 (resp. W_1) by approximation applies to $MM_{2\text{-}3}$ (resp. $WW_{2\text{-}3}$).

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