ON THE GEOMETRY OF OBSERVATION

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1. The crux of the matter

This is a report on work in progress; it is incomplete. The motivation for this study is a coincidence that this writer has found absolutely 'mystical,' and which seems to be very significant. It is, that the Lorentz group arises in structures that are not nowadays thought to be related.

The Lorentz structure first appeared in the famous transformations under the same name that arose in Special Relativity Theory. In that venue, they are considered to transform coordinates when the observer is thought to transform from one inertial frame to another. Alternately, they are applied to situations in which it is considered that the observer is fixed, but the object of interest is accelerated or "boosted" into another inertial frame. These applications are the traditional ones, and very well known.

Somewhat less well known, is that the Lorentz group arises also in the study of 'squeezing where the degree of "squeezing" is also specified by a Lorentz transformation. [1] Likewise, the Lorentz group is useful in describing a chain of optical devices. [2] The Lorentz group also arises naturally in analysis of the "entanglement" of qubits. [3] Finally, nonrelativistic analogues of the salient physical effects derived from Lorentz transformations, i.e., time-dilation and space-contraction, are observed in the mechanics of dislocations in crystals. [4, 5]

These applications have no obvious connection to a change of inertial frame— whatever they do have in common. But, under it all, there must be some common structure or logic that requires the Lorentz group for encoding. What is it?

2. The clue

There is one other venue in which the Lorentz group structure arises and which might provide the clue needed to find the answer to the question just posed. It is that the structure of projective geometry on $3 + 1$ Galilean space induces the Lorentz-Minkowski structure. [6] Further, geometrical projection is intuitively the structure of observation. Thus, it appears, in other words, that Special Relativity can be interpreted as the geometry of observation. This means that while the geometry of ontology is Galilean; it is the act of observing using light which realizes the hypothetical assumptions of projective geometry in $3 + 1$ space, where the "1" is a time dimension, and which induces the Minkowski-Lorentz structure.

Projection on $3 + 1$ space is not the simple projection on 3-space; it silently incorporates specific physically relevant hypotheses. One seems to be, that projection rays are straight; another is, that the ratio of any one of the spacial directions to the time dimension is a constant. These two features are easily recognized as parallel to the fundamental assumptions for Special Relativity. The main difference is that, in this projection formulation, it is clear that the ontological space of the source of projection rays is a Galilean space. This feature seems to be unknown or unrecognized in the standard presentations of Relativity.

It is also clear that the projection rays are in fact electromagnetic 'interactions.' It is worth considering just what such interactions might comprise. [7] Historically, there are two main notions or paradigms: particle beams and waves. Neither of these two, however, really corresponds fully to the character of the projection ray in the sense that each requires considerable hypothetical input to specify its character. On the other hand, the direct-interaction on the light cone conception, as introduced first (apparently) by Schwarzschild, requires virtually no additional definition: it seems tailor-made as a model for projection rays. [8] For these reasons, herein it shall be taken that the native electromagnetic interaction is in fact just such a projection ray from source charge to sink charge, where the latter can be in possession of either a human observer or his object of interest.

Clearly, an "observer" in the sense of an experimenter is not just an elementary sink charge, because, as a sentient being, he has two eyes and can triangulate. A single eye, on the other hand, can only determine the elevation and azimuth of an incoming ray. In the most primitive circumstance, such a ray should be conflateable with what is in field theory, the electric field (i.e., according to Coulomb's Law) and manifest itself as an attraction or repulsion to the source charge along the relative direction to the delayed position of the source charge.

3. The mathematics

All of the structure mentioned above in connection with my speculations on the fundamental nature of electric interaction as a physical realization of projection have been thoroughly worked out mathematically, however, not for the special purpose of just describing this interaction. [9–11] Thus, the logical development, or construction, of the this mathematical structure was so discovered, or created, so as to be optimal for purely mathematical argumentation, not its application to any physical phenomenon.

The key mathematical concepts center on the fact that the "celestial sphere" of any observer can be identified with the Riemann sphere on the Argand (complex) plane. [11] All any observer can discern regarding an incoming signal at an instant, is its azimuth and elevation, but neither the distance to the source nor the time of flight of the emission. In other words, the observer cannot distinguish between two signals (here meant to be momentary pulses with a given azimuth and elevation): one originating a relatively short time ago and at relatively close range, or one emitted from further away at a more remote time. That is, the *resolution* distance in the eye of the observer as far as he can tell, is 'zero' for these two pulses. Now, what I wish to argue here, is that it is no accident that this physical fact maps onto the Minkowski statement that there is no 'distance' on the light cone. But, according to the common and nowadays orthodox understanding attributed to this statement, the story has at least one chapter in which it is asserted that a traveler on the light cone, i.e., one moving with the speed of light experiences no flow of time. [12] From the view point taken herein, however, the physical situation is quite different: namely, that although the sources of signals 'on the light' cone are both spatially and temporally separate, an observer or recipient of these pulses is geometrically unable to distinguish either the spacial or temporal intervals separating them. Ontologically, that is, while these two sources maintain their separate identity, an observer circumstantially situated at the apex of the light cone cannot perceive these intervals; in other words, for him as only an epistemological matter, their true ontology remains indiscernible. These null distances are in fact the "resolution distances" only for particularly situated observers. From our viewpoint a light wave front takes a time interval of d/c to traverse the Euclidean distance d.

4. Philosophy

Further, the mathematics of Riemann spheres covers the issue of conformal transformations of the sphere. It turns out that they are the bilinear maps which leave the Minkowski metric (in our terms: the 'resolution interval') invariant. This structure is well known (albeit mostly to specialists), and it is understood that these transformations of the Riemann sphere take account of what, in physical terms, is the Bradley-aberration effect of incoming rays as seen by the 'eye' at the center of the Riemann sphere. Thus, those Lorentz transformations corresponding to simple spacial rotations yield the expressions for the same unaltered rays as seen by an 'eye' rotated with respect to the orientation originally considered. In the same manner, the Lorentz transformations corresponding to 'boosts' yield the coordinates of the same rays as seen by an eye moving with velocity v through the exact same momentary observation point as the stationary eye. The relative distortion of the moving eye's Riemann sphere takes the Bradley aberration due to his motion into account; it does not represent any physical change whatsoever of the source charge or of the momentary interaction-ray from it to the 'eye,' only its appearance to observers.

This understanding is at sharp variance to much contemporary understanding of Minkowski structure. In this projection viewpoint, all results of Lorentz (more generally of conformal) transformations do not affect the source, or even the 'sink,' i.e., the 'eye,' but just its perception. In plain text: Lorentz-FitzGerald contraction and time dilation are epistemological phenomena, that is, artifacts of the geometry of observation, or even just passive reception by a charge of electric force from another charge. Further, as these effects are taken as mere artifacts of observation, they cannot accumulate as is supposed to occur in, e.g., the twin paradox. In this view, the traveling twin will turn out to be exactly as old as his stay-at-home sibling when his trip is finished, which is *not* true, however, of the reports sent while underway to his stationary sibling using electromagnetic signals.

5. Fallout

If the thesis presented herein is accepted, then various arguments to be found in the literature get considerable support for their logical underpinning. Those which I wish to emphasis here include arguments originally publicized by Dingle, Sachs and this writer. Each of these arguments can be, arguably, better understood in terms of the geometry of projection as outlined above.

5.1. **Dingle.** Herbert Dingle advanced a criticism of Relativity Theory in the 1930's and 1940's, which, in my distillation, can be captured as a syntactical argument. [13] It is this: the principle of relativity states that all inertial frames are equivalent. Thus, a particular frame in which a human "observer" finds himself, is equivalent to those frames of whatever objects he is observing. If it is supposed that he is observing various cosmic rays which are underway at very high velocities in various directions, then the rest frame of each of these rays, or particles (alpha particles, say), is equivalent to that of the human observer. Included in everything else, this means that the human observer from the view point of any of these cosmic rays is suffering time dilation and Fitz-Gerald contraction. The antinomy that Dingle brought attention to, is, that there are multiple particles, so that the observer should, according to the convectional understanding, be suffering multiple time dilations and spacial contractions. Of course, we humans are all such observers, whether consciously or otherwise, and see full well that that there is no such ambiguity in our clocks or rulers.

This experience can lead to no other conclusion but, that time dilations and FitzGerald contractions are simply artifacts of the observation, and not induced characteristics of the objects being observed themselves. They can be only space-time perspective effects, not modifications of any sort of the observed physical object. Any other interpretation in Dingle's terms makes goulash of the logic and syntax of the vocabulary of our languages.

5.2. **Sachs.** Mendel Sachs, one might say, has rendered Dingle's argument in mathematical terms. That is, he demanded, where Dingle sought syntactical self consistency, mathematical consistency, but in a round-about way. [14]

Sachs argued that in the end the total situation involving the travel of the twin, because of accelerations, must be analyzed using General Relativity (GR). In this regard, Sachs calls on his quaternion version of GR, and argued that its equations should govern the whole trip, including the accelerations involved in launching the traveler, effecting his turn-around and finally the deceleration putting him back into the inertial frame of the stay-at-home twin. Sachs' point is that these equations in quaternion form are fully analytical (mathematically) and single-valued so that integrals of such functions will be path-independent. This implies that there will be no asymmetric aging, or that the path length of two world lines that cross twice will be of equal length between crossings, regardless of the specific paths.

Sachs' arguments seemed to have had little impact; perhaps because they were based on his quaternion formulation of relativity, and few seem to be confident in its full veracity. However, Sachs' argument can be reformulated in terms of a trip without accelerations. The idea is to consider a trip composed of two parts, each so configured that all accelerations are accomplished outside the actual segments of the trip of interest. That is, the outward bound portion involves a preaccelerated traveler who starts the clock of the stay-at-home twin as he passes. Then, he starts by touch-tag the clock of another inbound preaccelerated traveler at the turn-around point, who finally stops both his clock and the stay-at-home's clock again by just momentary contact. Standard analysis of just those portions of the trips between the contacts without any accelerations still leads to the conclusion that the total travel time involved is contracted. In this case the world lines are straight, taken as arc-length their expression is analytical and satisfy Sachs' argument.

5.3. **Geometry.** By a circuitous and fortuitous accident, this writer came up with a geometrical equivalent of Sachs' argument. It is best rendered graphically. [15] The essential point is that for diagramming the trip in its idealized version without accelerations, a crucial issue is that the actual location of the turn-around point for the traveler is the vital factor; see Fig. 1. If a specific object is identified as the marker for the turn around point, then this object will have a world line,

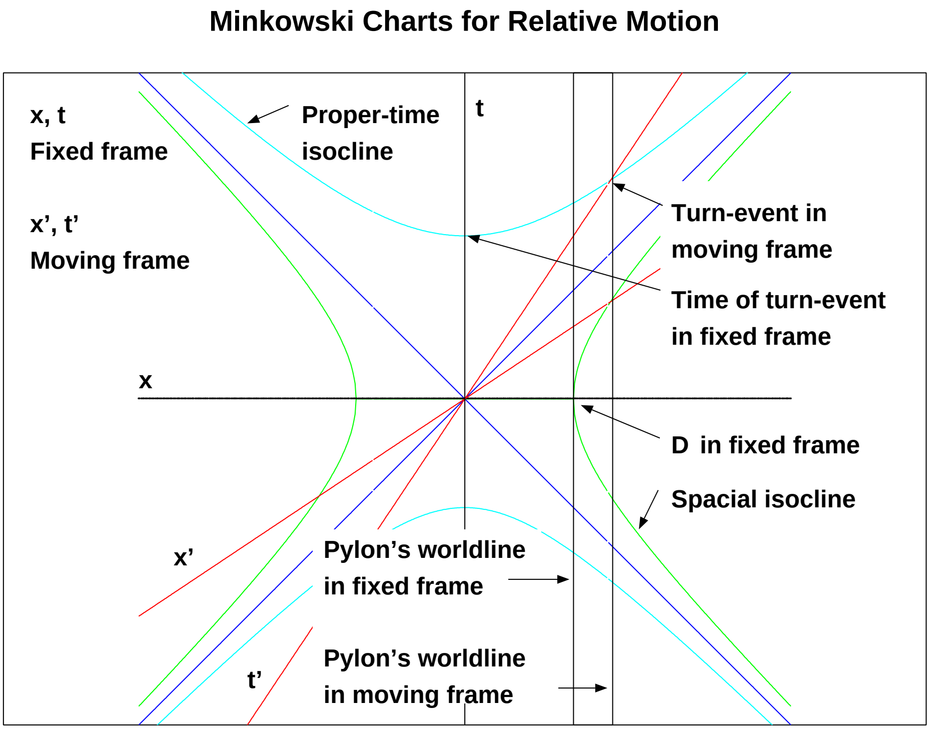

FIGURE 1. This figure is comprised of two Minkowski charts superimposed on each other. The world line of the turnaround point in the fixed frame passes through the point D on the x-axis. The corresponding point on the x'-axis is found by sliding up the eigenlength isocline to the intersection with the x'-axis. The world line of the pylon passes through this point on the prime chart. The intersection of the pylon's world line with the t'-axis is the point on the traveler's chart representing the 'turn-around' event. The eigentime of the turn-around event in the fixed frame is found by sliding down that eigentime isocline which passes through the turn-around event to its intersection with the t-axis. It is clear that this value is identical with the time assigned by the fixed twin to the turn-around event as it may be projected horizontally over to the intersection of the pylon's world line in the fixed frame with the time axis of the traveler. The twin paradox arises by using, incorrectly, that eigentime isocline which passes through the intersection of the traveler's and the pylon's fixed frame world-lines.

but the position of the world line on the Minkowski chart of the traveler does not coincide with the same world line on the chart of the stay-at-home. Taking this little complication into account, is essentially equivalent to respecting Sachs' point regarding self-consistency of the relevant mathematics, and of Dingle's point the syntactical self-consistency of the words used to discuss both the ontology and its mathematical rendition.

6. Conclusions

In a certain technical sense, the arguments as rendered herein are fully adequate to support the conclusion, that asymmetric aging is simply an artifact of the structure of the act of observation itself. This suffices arguably to resolve the twin paradox as such, but is evidently insufficient to overcome all the accumulated folklore.

Moreover, there is a complication, with respect to application of these ideas to the mechanics of interacting massive charged particles. It is, that each particle is effectively an "observer" of all the others, necessitating the incorporation of the attendant mathematical machinery into the coupled equations of motion of the particles. While this complication appears to be contained by the fact that the relativistic Lagrangian for interacting particles has a Dirac delta function with the argument being the Minkowski metric, [16] it is not clear, that the subsequent manipulations automatically take full account of all the complications which are due to the noncommutivity of Lorentz boosts. This is a question for further analysis. To test this matter, it is the writer's intention to program the formulas used, to encode the mappings from the celestial sphere to the Lorentz group on the Argand plane and apply them to a specific problem. The best candidate for such a problem may be the Sagnac Effect, as it is still the topic of much controversy.

In any case, the extraordinary applications of the Lorentz group can also be understood better already as encodifications of the aberrations of light in consequence of the geometry of observation. That Lorentz transformations describe optical devices, for example, can be seen a consequence of the fact that optical devices are deliberate means of altering the apparent direction of light rays, which is operationally equivalent to the same sort of aberrations due to the motion of an observer.

Finally, it is the writer's intention to attempt to re-derive the relativistic formalism with arguments parallel to those used in the mathematical development. The purpose of such an exercise would be to precisely identify the physical hypothetical inputs and use them to motivate the corresponding mathematical hypothetical inputs. With this, hopefully, counterintuative relativistic effects will be attributed directly and logically to hypothetical inputs for which there is undisputed empirical evidence, so that any dispute over the true nature of astonishing effects (i.e., asymmetric aging and dilation) can be resolved.

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